

*DRAFT*



## **Dynamic Econometric Loss Model**

A Default Study of US Subprime Markets

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### **Abstract**

The meltdown of the US subprime mortgage market in 2007 triggered a series of global credit events. Major financial institutions have written down approximately \$120 billion of their assets to date and yet there does not seem to be an end to this credit crunch. With traditional mortgage research methods for estimating subprime losses clearly not working, it now requires revised modeling techniques and a fresh perspective of other macro-economic variables to help explain the crisis. During the subprime market rise/fall era, the Housing Price Index (HPI) and its annual appreciation (HPA) had been the main blessing/curse attributed by researchers. Unlike the traditional models, our Dynamic Econometric Loss (DEL) model not only applies the static loan and borrower characteristic variables such as loan terms, Combined-Loan-To-Value ratio (CLTV), Fair Isaac credit score (FICO), as well as dynamic macro-economic variables such as HPA to projects defaults and prepayments, but also applies the spectrum of delinquencies as an error correction term to add an additional 15% accuracy to the model projections. In addition to our delinquency attribute finding, we find that cumulative HPA and the change of HPA contribute various dimensions that greatly influence defaults. Another interesting finding is a significant long-term correlation between HPI and disposable income level (DPI). Since DPI is more stable and easier for future projections, it suggests that HPI will eventually adjust to coincide with DPI growth rate trend and that HPI could potentially experience as much as a 14% decrease by the end of 2009.

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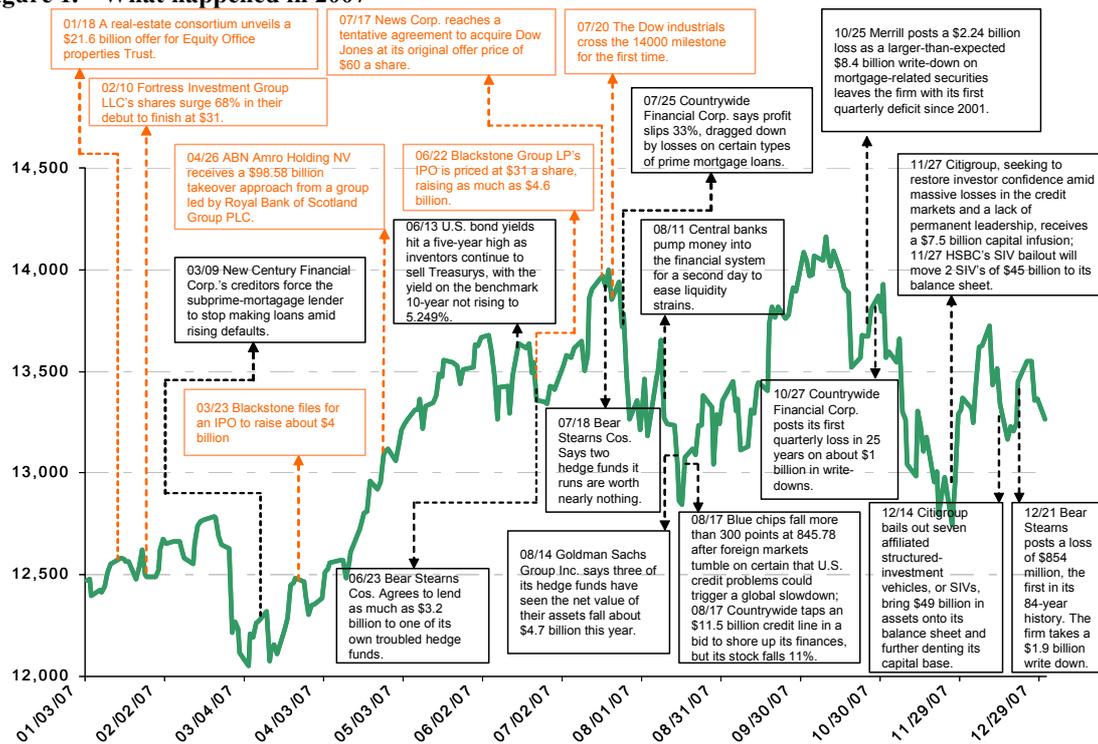
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## INTRODUCTION

Subprime mortgages are made to borrowers with impaired or limited credit histories. The market grew rapidly when loan originators adopted a credit scoring technique such as FICO to underwrite their mortgages. A subprime loan is characterized by a FICO of between 640 to 680 or less, versus the maximum of 850. In the first half of the decade, the real estate market boom and well-received securitization market for deals including subprime mortgages pushed the origination volume to a series of new highs. In addition, fierce competition among originators created various new mortgage products and a relentless easing of loan underwriting standards. Borrowers were attracted by the new products such as “NO-DOC, ARM 2/28, IO” that provided a low initial teaser rate and flexible interest-only payments in the first two years, without documenting their income history.

As the mortgage rates began to increase in the summer of 2005 and housing activity revealed some signs of a slowdown in 2006, the subprime market started to see some cracks as the delinquencies began to rise sharply. The distress in the securitization market backed by subprime mortgages and the resulting credit crisis, had a ripple effect initiating a series of additional credit crunches. All this has pushed the U.S. economy to the edge of recession and is jeopardizing the global financial markets.

Figure 1. What happened in 2007



The rise and fall of the subprime mortgage market and its ripple effects raises a fundamental question. How can something as simple as subprime mortgages, which

accounts for only 6-7% of all US mortgage loans, be so detrimental to the broader economy as well as the global financial system?

Before formulating an answer to such a big question, we need to understand what the fundamental risks of subprime mortgages are. Traditional valuation methods for subprime mortgages are obviously insufficient to measure the associated risks that triggered the current market turmoil. What is the missing link between traditional default models and reality? Since a mortgage's value is highly dependent on its future cash flows, the projection of a borrower's embedded options becomes essential to simulate its cash flows. Studying consumer behavior to help project prepayments and defaults (call/put options) of a mortgage is obviously the first link to understanding the current market conditions.

This paper focuses on modeling the borrower's behavior and resultant prepayment or default decision. A Dynamic Econometric Loss (DEL) model is built to study subprime borrower behavior, and project prepayment and default probabilities based on historical data from Loan Performance's subprime database (over 17MM loans) and prevailing market conditions from 2000 to 2007.

The paper is organized in the following manner. We started by constructing a general model framework in a robust functional form that is able to not only easily capture the impact of individual model determinants, but also be flexible enough to be changed to reflect any newly found macro-economic variables. We then modeled default behavior through an individual factor fitting process. Prepayment modeling follows a similar process with consideration of the dynamic decision given prior prepayment and default history. The delinquency study builds the causality between default and delinquencies and the relationship within the spectrum of different delinquencies. We then utilized the delinquencies as a leading indicator and error correction term to enhance the predictability of the forecasted defaults by 15%. Our findings and forthcoming research are then drawn in conclusion section.

## MODEL FRAMEWORK

When a lender issues a mortgage loan to its borrower, the loan is essentially written with two embedded American options with an expiration co terminus with the life of the loan. The lender will then receive payments as compensation for underwriting the loan. The payments will include interest, amortized principal and voluntary/involuntary prepayments along with any applicable associated penalties. The risk for the lender is they might not receive the contractual payments and will need to go after the associated collateral to collect the salvage value of the loan. Additionally, the foreclosure procedure could be costly and time consuming.

Unscheduled payments come in two forms. A voluntary prepayment is usually referred to simply as ‘prepayment’ and an involuntary prepayment, which is known as ‘default’ (with lags to potentially recover some portion of interest and principal proceeds). Prepayment is nothing but a call option on some or all of the loan balance plus any penalties at a strike price that a borrower has the right to exercise it if the option is in-the-money. By the same token, default is a put option with the property’s market value as the strike price to the borrower. Understanding the essence of both options, we need to find the determining factors that trigger a borrower to prepay/default through filtering the performance history of the loan. A list of determinant factors regarding consumer behavior theory for modeling default and prepayment will be discussed in the next two sections.

In order to construct a meaningful statistical model framework for empirical work, the availability of data and the data structure are essential. In other words, our model framework is designed to take full advantage of data structure of the Loan Performance subprime mortgage historical information and the prevalent market information. The model empirically fits to the historical default and prepayment information of US subprime loan performance from 2000 to 2007 (over 17MM loans).

**Figure 2. Number of Securitized Alt-A and Subprime Mortgage Origination**

Type / Orig. Year	ARM OTHER	ARM2/28	ARM3/27	ARM5/25	FIXED	Grand Total
2000	11,452	187,232	68,430	4,059	390,671	661,844
2001	11,389	261,316	67,018	10,449	477,718	827,890
2002	33,776	434,732	100,939	25,827	605,233	1,200,507
2003	51,548	697,073	164,228	71,839	958,170	1,942,858
2004	221,818	1,239,522	413,366	213,572	1,172,413	3,260,691
2005	496,697	1,577,003	393,020	301,829	1,619,257	4,387,806
2006	490,975	1,137,345	234,344	349,460	1,754,382	3,966,506
2007	99,946	161,480	36,795	160,549	404,278	863,048
Grand Total	1,417,601	5,695,703	1,478,140	1,137,584	7,382,122	17,111,150

Mathematically, our general framework constructs the default and prepayment rates as two separate functions of multiple-factors that the factors are categorized into two types – static and dynamic.<sup>1</sup> The static factors are initially observable when a mortgage is

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<sup>1</sup> There is no industry standard measure for default rate, thus a different definition on default rate will give a very different number. As there is no set standard we define our default rate based on the analysis in this

originated such as borrower's characteristics and loan terms. Borrower's characteristics include Combined-Loan-To-Value ratio (CLTV), Fair Isaac credit score (FICO), and Debt-To-Income ratio (DTI). Loan terms include loan maturity, loan seasoning, original loan size, initial coupon reset period, interest only (IO) period, index margin, credit spread, lien position, documentation, occupancy, and loan purpose. The impacts to the performance of a loan from the static factors provide the initial causality impacts yet their influence could diminish or decay as the information is no longer up to date.

Dynamic factors include several macro-economic variables such as Housing Price Appreciation (HPA), prevailing mortgage interest rates, consumer confidence, gross disposable income, employment rate, and unemployment rate. These dynamic factors supply up to date market information and thus play an important role in dynamically capturing market impacts. The accuracy of capturing causality impacts due to the static factors and the predictability of the dynamic factors presented a constant challenge during the formulation of this model.

For each individual factor, a non-linear function is formulated according to its own characteristics. For example, a "CLTV" factor for modeling default is formulated as the function of default rate over CLTV ratio. However, a DOC factor is formulated as the function of multiplier over discrete variables of "FULL" versus "LIMITED" with percentages of respective groups.

A general linear function of combined multi-factor functions is then constructed as basic model framework to fit the empirical data and to project forecasts for prepayments and defaults.<sup>2</sup> In the following sections, we will discuss each factor in detail.

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paper, "Loss Severity Measurement and Analysis", The MarketPulse, LoanPerformance, 2006 Issue 1, 2 – 19. Please refer to Appendix I for definition of default we used throughout this paper.

<sup>2</sup> See Appendix II for the details of model specification.

## DEFAULT MODELING

### Default Modeling Factor Components:

Seasoning	Occupancy
Combined Loan-To-Value (CLTV)	Owner
Credit Score (FICO)	Second home
Debt-To-Income ratio (DTI)	Investor
Payment Shock (IO)	Property Type
Relative Coupon Spread	Single-Family
Loan Size	Multi-Family
Lien	Condo
First	Loan Documentation
Second and others	Full
Loan purpose	Limited
Purchase	Housing Price Appreciation (HPA)
Refinance	State Level
Cashout	CBSA Level

### Seasoning

Loan information regarding borrower’s affordability is usually determined at origination. As a loan seasons, its original information decays and its default probability starts to surge. A seasoning baseline curve with annualized Constant Default Rate (CDR) against its seasoning age would post a positive slope curve for the first three years.

Figure 3. Seasoning: CDRs by Date and Vintages of ARM 2/28

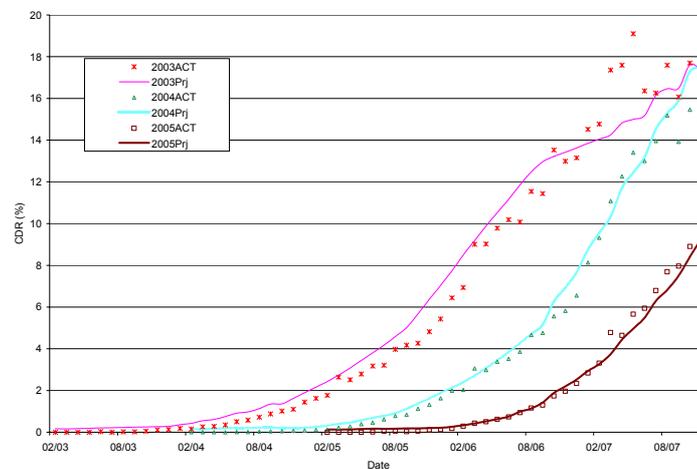


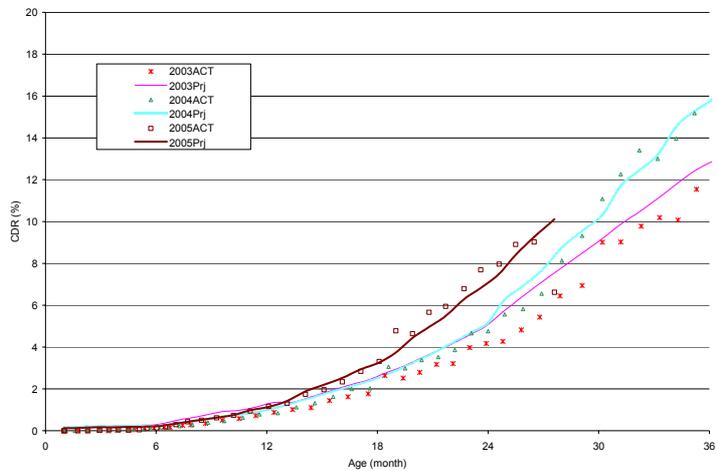
Figure 3 shows actual CDR curves and their fitted result of different vintages of ARM 2/28 mortgage pools. They roughly follow a similar shape to the Standard Default Assumption (SDA) curve.<sup>3</sup> But as shown in the figure 4, the ramp up curve can be very different for different vintages.

<sup>3</sup> SDA is based on Federal Housing Administration (FHA)’s historical default rate and was developed by Bond Market Association (BMA), now known as Securities Industry and Financial Markets Association (SIFMA).

### Why is the 2005 seasoning pattern faster than prior vintages?

Since the seasoning baseline curve is not independent of dynamic factors, a dynamic factor such as HPA could tune vintage seasoning curves up and down. In Figure 4, the 2005 seasoning pattern is significantly steeper than its prior vintages. Looser underwriting standards, deteriorating credit fundamentals can be an important reason. The negative HPA obviously starts to adversely impact all vintages after year 2005.

**Figure 4. Seasoning: CDRs by Age and Vintages of ARM 2/28**



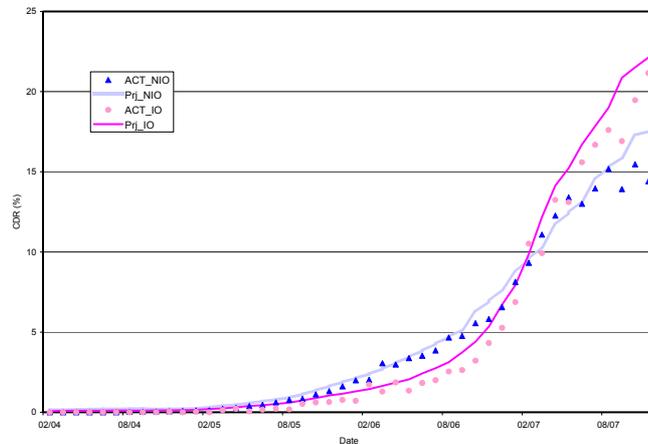
Source: Beyondbond Inc, LoanPerformance

### **Payment Shock – Interest Only (IO)**

The boom of subprime market in recent years has introduced new features to the traditional mortgage market. An ARM 2/28 loan with 2-year Interest Only (IO) feature has a low fixed initial mortgage rate and also pays no principal for the first two years prior to the coupon is reset<sup>4</sup>.

When the IO period ends, the borrower typically faces a much higher payment based on its amortized principal plus the fully indexed interest. This sudden rise in payments could produce a ‘Payment Shock’ and test the affordability to borrowers. Without the ability to refinance borrowers who are either under a negative equity situation or not able to afford the new rising payment will have a higher propensity to default. Consequently, we see a rapid surge of default rates after the IO period.

**Figure 5. IO Payment Shock: CDRs by Date of ARM 2/28**



<sup>4</sup> The reset is periodical and the interest rate is set as Index + Margin.

The ending of the IO period triggers payment shock and will manifest itself with a spike to delinquency.<sup>5</sup> Delinquent loans eventually work themselves into the defaulted category within a few months after the IO period ends. Figure 5 shows the difference patterns and the default lagging between IO and Non-IO of ARM 2/28 pools.

### **Combined Loan-to-Value (CLTV):**

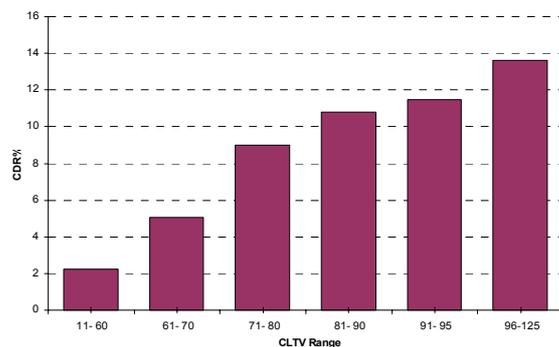
LTV ratio measures the ratio of mortgage indebtedness to the property’s value. When multiple loans have liens added to the indebtedness of the property, the resulting ratio of CLTV becomes more meaningful measure of the borrower’s true equity position.

However, the property value might not be available if a “market” property transaction does not exist. A refinanced mortgage will refer an ‘appraisal value’ as its property value. Note that ‘appraisal value’ could be manipulated during ferocious competition amongst lenders in a housing boom market and would undermine the accuracy of CLTV.

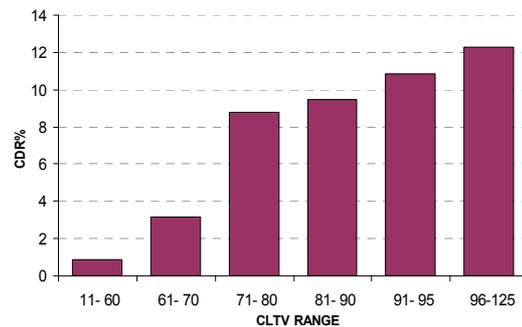
As we know, default is essentially a put option embedded to the mortgage for borrower. In a risk neutral world, a borrower should exercise the put if the option is in-the-money. In other words, a rational borrower should default the mortgage if CLTV is greater than one or the borrower has negative equity.

**Figure 6. Stratified seasoned CDR over CLTV ranges**

*CDR vs. CLTV of ARM 2/28 Non-IO with age >24*



*CDR vs. CLTV of ARM 2/28 Non-IO with age >24 and FICO between 641 and 680*



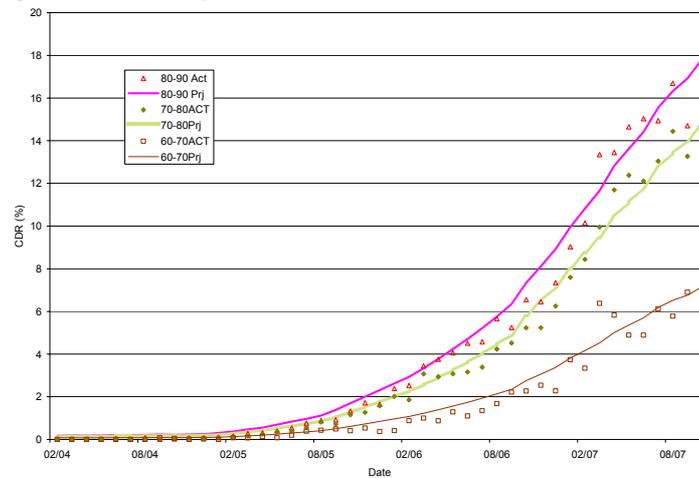
Source: Beyondbond Inc, LoanPerformance

At higher CLTVs it becomes easier to reach a negative equity level as loan seasons and its default probability increases. Figure 6 provides the actual stratification result of CDR over various CLTV ranges. Obviously, CDR and CLTV are positive correlated. In addition, lower CDR values are observed for top subprime tier FICO ranged from 640 to 680. It shows that the FICO tier granularity is another important factor in modeling.

<sup>5</sup> The delinquency rate is measured by OTS (Office of Thrift Supervision) or MBA (Mortgage Bankers Association) convention. The difference between these two measures is how they count missed payments, MBA delinquency rate count the missed payment at the end of the missing payment month while OTS delinquency rate count the missed payment at the beginning of the following month after missing payment. This difference will pose a 1-30 days delay of record. OTS delinquency rate is the prevailing delinquency measure in subprime market.

However, since CLTV is obtained at the loan's origination date, it does not dynamically reflect housing market momentum. We introduce a dynamic CLTV that includes housing price appreciation from loan origination to attempt to estimate more precisely the actual CLTV. This dynamic CLTV allows us to better capture the relationship between CLTV and default. Figure 7 clearly illustrates that different CLTV groups show a different layer of risk level.

Figure 7. CDRs by Date and CLTVs of ARM 2/28



Source: Beyondbond Inc, LoanPerformance

## FICO

FICO score is an indicator of borrower's credit history. Borrowers with high FICO scores maintain a good track record of paying their debts on time with a sufficiently long credit history.<sup>6</sup>

In recent years, people believe FICO is no longer an accurate indicator due to the boom of hybrid ARM loans and fraudulent reporting to the credit bureaus. Since refinancing is much easier to obtain than before, issuers are giving out tender offer to borrowers in order to survive the severe competition amongst them.

CLTV and FICO score are two common indicators that the industry uses to predict default behavior.<sup>7</sup> We examine the combined CLTV and FICO effects on CDR as shown in the Figure 8. The figure presentation a same 3-D surface of stratified CDR rates over CLTV and FICO ranges from two different angles for seasoned ARM 2/28 pools. The relationship between CLTV and CDR is positively correlated across various FICO ranges. However, the relationship between FICO and CDR is somewhat negatively correlated across various CLTV ranges. However, the case is not as significant. FICO factor impact is obviously not as important as we originally expected.

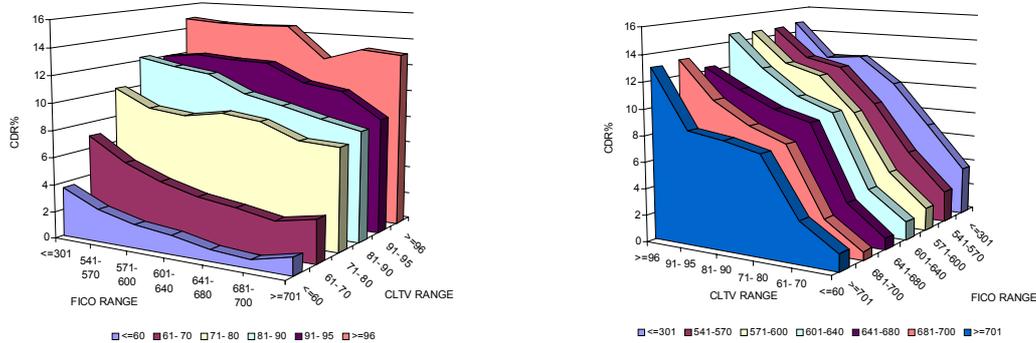
<sup>6</sup> According to Fair Isaac Corporation's (The Corporation issued FICO score measurement model) disclosure to consumer, 35% of this score is made up of punctuality of payment in the past (only includes payments later than 30 days past due), 30% is made up of the amount of debt, expressed as the ratio of current revolving debt (credit card balances, etc.) to total available revolving credit (credit limits) and 15% is made up of length of credit history. Severe delinquency (30 plus) and credit history length make up 50% of the FICO score. This score reflects people's willingness to repay. It's essentially the probability distribution for people's default activity on other debts such as credit card and/or utility bills and etc. Statistically speaking, people with higher FICO score will have lower probability to default.

<sup>7</sup> Debt-to-Income ratio is also an important borrower characteristic, but in recent years, more Limited-Doc or/and No-Doc loans are issued. For these loans, many of them do not have DTI ratio report, so we consider DTI separately for different DOC type.

**Figure 8. Stratified CDR by CLTV and FICO of ARM 2/28**

CDR vs. FICO and CLTV of Seasoned ARM 2/28

CDR vs. FICO and CLTV of Seasoned ARM 2/28

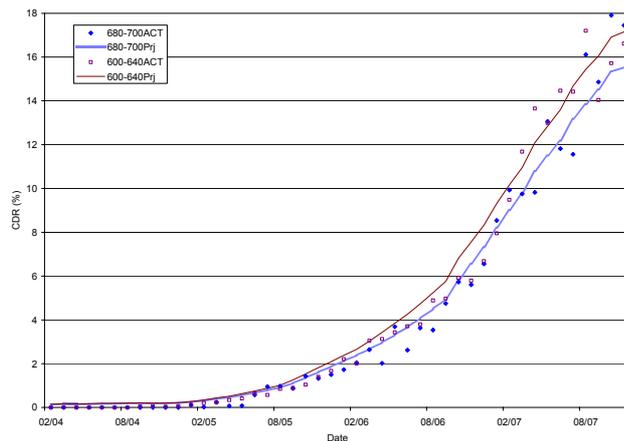


Source: LoanPerformance

In our analysis, CLTV=75 and FICO=640 serves as a base curve, and then we adjust the CDR according to movements of other default factor.

Figure 9 gives an example of fitting results based on ARM2/28 2004 vintage pools. The difference between 600-640 and 680-700 FICO ranges only make approximately a 1% difference in CDR for a seasoned pool.

**Figure 9. FICO: CDRs by Date and FICOs of ARM 2/28**



Source: Beyondbond

### **Debt-To-Income Ratio (DTI) and Loan Documentation (DOC)**

The DTI in this paper is defined as the back-end DTI which means the debt portion for calculating the DTI ratio includes not only PITI (Principal + Interest + Tax + Insurance) but also other monthly debts such as credit card payments, auto loan payments and other personal obligations.<sup>8</sup> The DTI ratio shows the affordability of a loan to a borrower and provides us with a clearer picture of a borrower with an exceptionally high DTI. For different regions of the country, the DTI ratio could imply a different financial condition

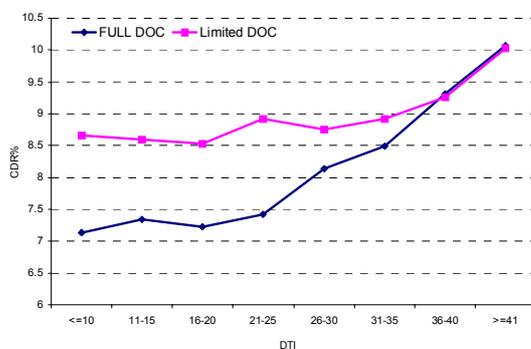
<sup>8</sup> There are two major measures of DTI in the industry, Front-End DTI ratio = PITI/Gross Monthly Income; Back-End-DTI ratio = PITI + Monthly debt/Gross Monthly Income. PITI=Principle + Interest + Tax + Insurance.

of the borrower because of different living standards and expenses between those of rural areas and large cities.

DTI is captured and reported as part of the loan documentation process. Loan documentation, also referred to as DOC, consists of three major groups: ‘FULL DOC’, ‘LOW DOC’, and ‘NO DOC’. Lenders usually require a borrower to provide sufficient ‘FULL’ documentation to prove their income and assets when taking out loans. People who are self-employed and/or wealthy and/or have lumpy income stream are considered as borrowers with ‘LIMITED’ (LOW or NO) documentation. In recent years, the fierce competition pushed lenders to relax their underwriting standards and originated more LIMITED DOC loans with questionable incomes. This uncertainty regarding income will pose uncertainties in determining the real DTI.

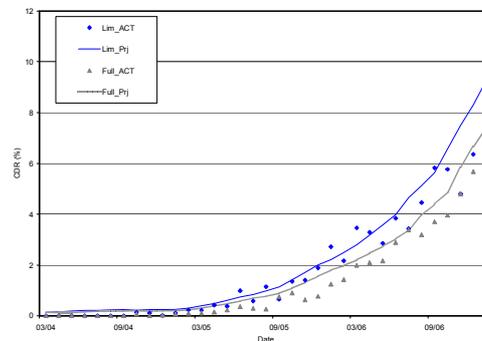
The stratification report shows two very different patterns of default between FULL and LIMIT documentation categories when analyzing the DTI effect. For FULL-DOC loans, default probability versus DTI is very much positively correlated, CDR increases as the DTI increases. Since FULL-DOC loans are loans that have documented income and assets, it shows the default DTI relationship most clearly as Figure 10. LIMITED-DOC has weaker relationship compared to FULL-DOC. Figure 11 shows the two different time series pattern of CDR curves and their fitted values between FULL and LIMITED DOCs.

**Figure 10. Stratified CDR by DTI ranges**  
Stratified CDR of seasoned pools between 2000-2007 by documentation types, FULL and LIMITED



Source: Beyondbond Inc, LoanPerformance

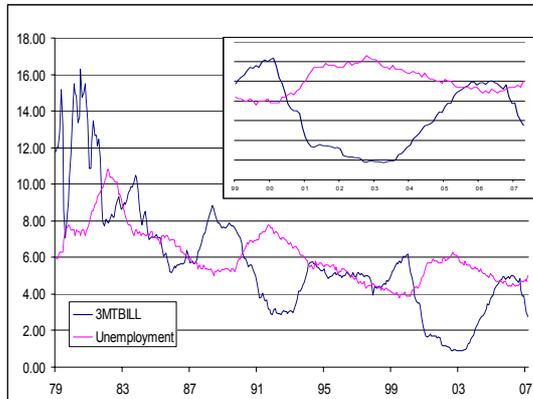
**Figure 11. DOC: Actual vs. Fitted for 2004**  
Actual versus Fitted CDR curve over time by documentation types, FULL and LIMITED for 2004 vintages



Source: Beyondbond Inc, LoanPerformance

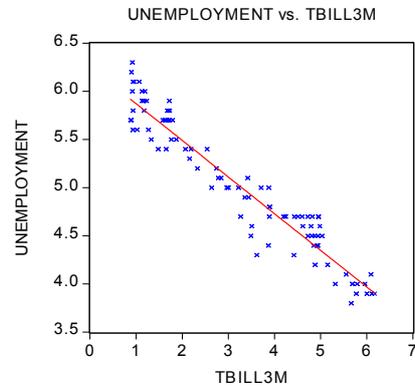
Since income is one of the main elements in determining the DTI ratio, the macro-economic variable, unemployment rate, becomes an important determinant that affects an individual’s income level. We find an interesting result when we plot the unemployment rate against 3-month U.S. Treasury Bills. They are very negative correlated for the last 7 years. Whether it was a coincidence or not, it suggested that the monetary policy has been mainly driven by the unemployment numbers.

**Figure 12. Unemployment Rate 1979-2007**



Source: US Bureau of Census

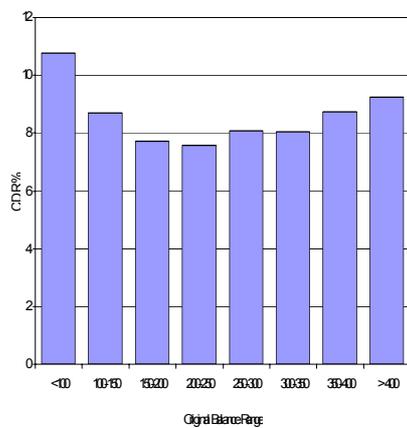
**Figure 13. Unemployment vs. T-Bill 3mo**



Source: US Bureau of Census

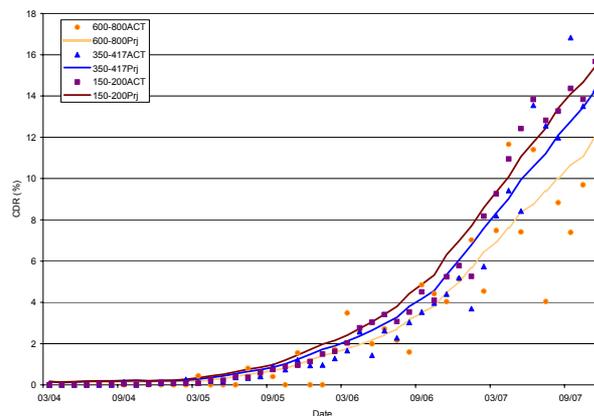
## Loan Size

**Figure 14. Loan Size Stratification**  
Seasoned CDR by different Loan Size ranges



Source: LoanPerformance

**Figure 15. Size: Actual vs. Fitted CDR for 2004**



Source: Beyondbond Inc, LoanPerformance

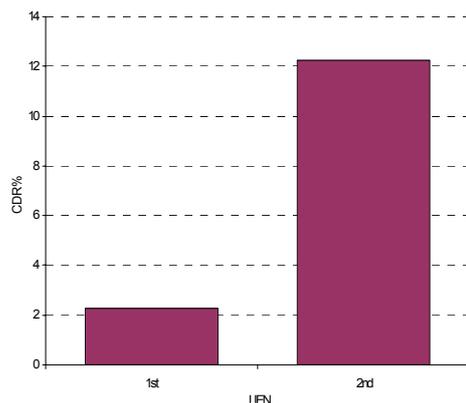
Is bigger better? The conventional argument is that larger loan size implies a better financial condition and lower likelihood of default. According to the stratification results based on original loan size in Figure 14, CDR forms a smile curve across original loan balance. Loans with sizes larger than \$350,000 tend to be a bit riskier although the increment is marginal. Loans with a size less than \$100,000 also seem riskier. Larger loans do not seem to indicate that they are better credits. The original loan size usually is harder to interpret as it can be affected by the other factors such as lien, property type, and geographical areas. For example, a \$300,000 loan in a rural area may indicate that a borrower with growing financial strength; while the same amount in a prosperous large city may indicate a borrower with weak purchasing power. Without putting size into the

context of property type and geographic location, the factor could be misleading. This may explain why we do not see a clear shape forming in Figure 14. Figure 15 shows the three different time series pattern of CDR curves and their fitted values based on their loan size ranges. Since the size is mixed for all the property types, the pattern and fitted results for each category is distorted and the fit is not as good as other factors.

## Lien

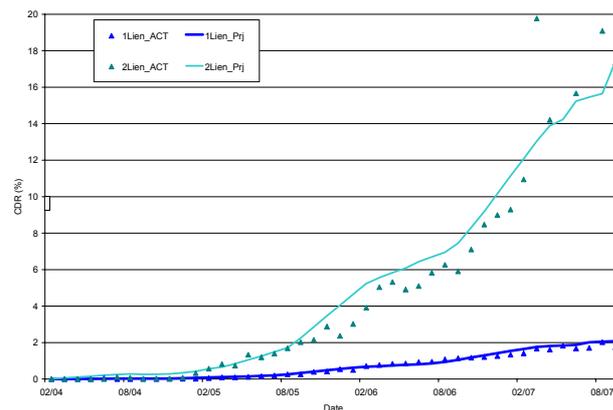
As we all know that the 2<sup>nd</sup> mortgage/lien has lower priority to the collateral asset than 1<sup>st</sup> lien mortgage/lien in the event of a default. Thus, the 2<sup>nd</sup> lien is riskier than the 1<sup>st</sup> lien. The 2<sup>nd</sup> lien borrowers usually maintain higher credit score, usually with a FICO greater than 640. We sometimes see a very mixed effect if the layer risk is not put into consideration. In Figure 16, 2<sup>nd</sup> lien loans are significantly riskier than 1st lien loans when measured against comparable FICO ranges for both liens. Figure 15 shows the three different time series pattern of CDR curves and their fitted values based on their liens.

**Figure 16. Lien Stratification**  
Seasoned CDR of 1<sup>st</sup> versus 2<sup>nd</sup> Liens



Source: LoanPerformance

**Figure 17. Lien: Actual vs. Fitted CDR for 2004**



Source: Beyondbond Inc, LoanPerformance

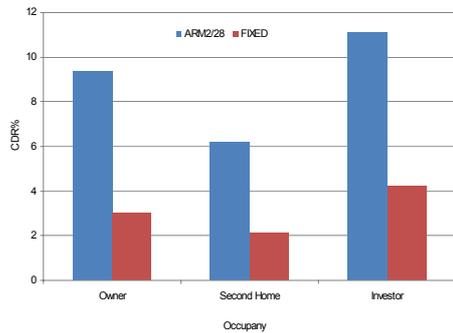
## Occupancy

Occupancy consists of three groups: ‘OWNER’, ‘INVESTOR’, and ‘SECOND HOME’. The ‘OWNER’ group views the property as their primary home, rather than as an alternative form of housing or an investment. This group will face emotional and financial distress if the property is in foreclosure or REO. Thus, this group has a lower propensity to default compared with others if all other factors remain the same. On the other hand, ‘INVESTOR’ and ‘SECOND HOME’ groups would be more risk neutral and are more willing to exercise their options rationally. In other words, they should have higher default risk.

Figure 18 reports an occupancy stratification regarding the default risk profile. The result evidently supports the risk neutral idea with respect to the ‘INVESTOR’ group and ‘INVESTOR’ does show the highest default risk among all three groups. The ‘OWNER’ group however, is not the lowest default risk group. Instead, the ‘SECOND HOME’

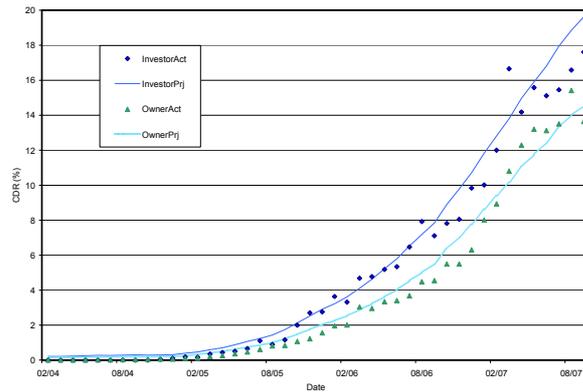
group is the lowest one. The observation is interesting, but not so intuitive. It indicates that when a borrower faces the financial stress, a ‘SECOND HOME’ will be sold first even at a loss to support his/her primary home. Thus the default risk of ‘SECOND HOME’ is actually reduced by incorporating a borrower’s primary home situation and cannot be simply triggered by the risk neutral idea. Figure 19 shows the two different time series pattern of CDR curves and their fitted values between ‘OWNER’ and ‘INVESTOR’.

**Figure 18. Occupancy Stratification**  
Season CDR by Occupancy types for ARM 2/28 and FIXED 2000-07 vintages



Source: LoanPerformance

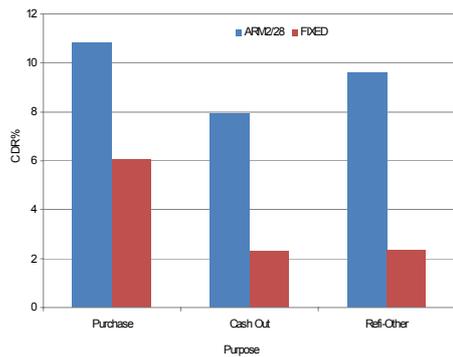
**Figure 19. Occupancy: Actual vs. Fitted CDR for 2004**



Source: Beyondbond Inc, LoanPerformance

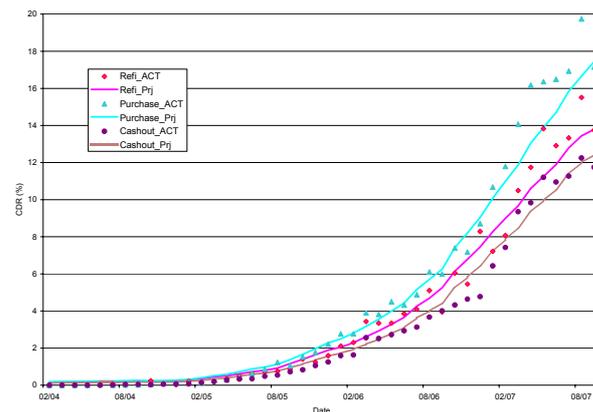
## Purpose

**Figure 20. Purpose Stratification**  
Season CDR by Purpose types for ARM 2/28 and FIXED 2000-07 vintages



Source: LoanPerformance

**Figure 21. Purpose: Actual vs. Fitted CDR for 2004**



Source: Beyondbond Inc, LoanPerformance

Loan Purpose classifies three key reasons for borrowing a loan as ‘PURCHASE’, ‘CASHOUT’, and ‘REFI’.<sup>9</sup> ‘PURCHASE’ means the borrower is a first time home

<sup>9</sup> For simplicity sake, we put refinance, 2<sup>nd</sup> mortgage, and other miscellaneous types as ‘REFI’.

buyer. ‘CASHOUT’ refers to a refinance loan with extra cash inflow to the borrower due to the difference between new increased loan amount and the existing loan balance. ‘REFI’ uses the loan for refinancing the outstanding balance without any additional funds draw from the equity in the property.

‘CASHOUT’ and ‘REFI’ usually reflects an intention to rollover the IO period or benefit from a lower mortgage rate. They can only be afforded by borrowers in good financial condition. ‘REFI’ is a group of borrowers with a higher FICO, LTV as compared to the other two categories. So we expect the ‘REFI’ Loans to have a lower default rate than ‘PURCHASE’ loans. The argument seems correct for the fixed rate mortgages. ‘REFI’ borrowers have much lower default probability than ‘PURCHASE’.

Beginning in 2007, the credit crunch hit the market and most of the lenders tightened their credit standards. Hybrid ARM types of loans, such as ARM 2/28, facing new resets, borrowers who no longer qualified for refinancing were in danger of defaulting. If these people can no longer afford the payment after IO and/or reset, they will eventually enter default. ARM 2/28 loans show a significant increase in defaults for ‘REFI’ purpose as compared with FIXED rate loans. Figure 21 shows the three different time series pattern of CDR curves and their fitted values among various purpose types.

### ***Dynamic Factors: Macro-Economic Variables***

As we have mentioned in Model Framework, macro-economic variables such as HPI, interest rate term structure, unemployment rate, and etc. that supply up to date market information can dynamically capture market impacts.

In theory, an economy generally maintains its long-term equilibrium as “Norm” in the long run and a handful of macro-economic variables are usually used to describe the situation of the economy. While the economy is in its “Norm” growing stage, these macro-economic variables usually move or grow very steadily and the risk/return profile for an investment instrument can be different depending on its unique investment characteristics. Because of that, a diversified investment portfolio can be simply constructed based on relationship of the correlation matrix. Thus the macro-economic variables usually are ignored during the “Norm” period. However, when an economy is under stress and approaches a “bust” stage, many seemingly uncorrelated investments sync together. The same the macro variables become the main driving forces that crucially and negatively impact the investment results. The current credit crunch is creating mark to market distress for investments across not only various market sectors but also credit ratings, clearly describing our view regarding to these macro-economic variables.

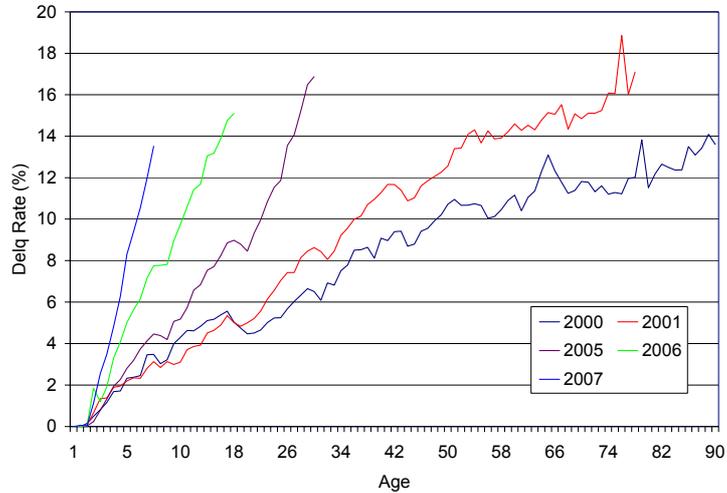
Since the severe impacts due to these variables mostly occur in economic downturn, cross correlation could provide a preliminary result in understanding the causality and the magnitude of their relationship. The dynamic interaction between these variables and consumer behavior would then provide a better sense of prediction and therefore either prevent the next downturn or efficiently spot an investment opportunity based on the next market recovery.

## Housing Price Appreciation (HPA)

The Housing Price Index (HPI) has been the most quoted macro-economic variable that causes the high delinquency and default rates since the beginning of subprime crisis.<sup>10</sup> Thus Housing Price Appreciation (HPA) which measures the housing appreciation rate compared with a year earlier has become the most important indicator within the U.S. housing market. By comparing the 30-day delinquency across vintages, the delinquency rates unidirectionally increase after the 2005 vintage.

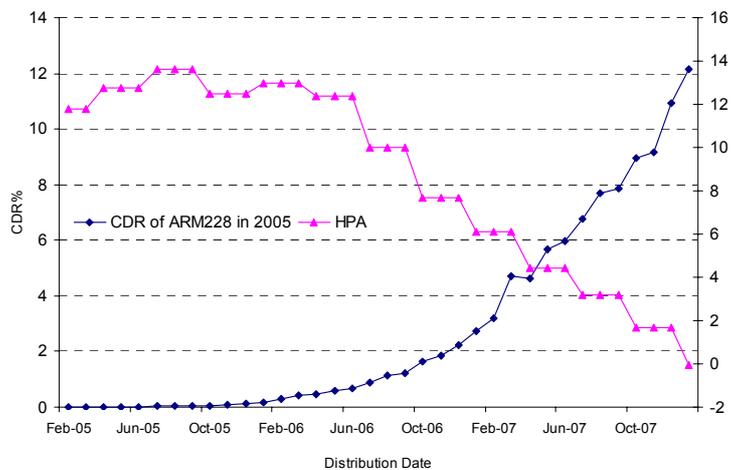
When we look at our seasoning pattern across 2000-2005 vintages, the 2005 seasoning pattern starts to surge after 18-month of age or the 3<sup>rd</sup> quarter of 2006. Coincidentally, HPA started to decline in the 2<sup>nd</sup> quarter of 2006. Although a similar HPA pattern appeared at the 3<sup>rd</sup> quarter of 2003, the main difference was that the former one was the up-trend of HPA, but the latter was on a down-trend. Defaults in the 2003 were obviously lower than in 2006 with comparable loan features and seasoning/age. In order to capture this subtle trend difference, we studied HPI and its various dimensions in addition to HPA level which proved illustrative.

Figure 22. 30-day Delinquency of ARM2/28 by Vintages



Source: OFHEO, LoanPerformance

Figure 23. HPA versus CDR of ARM2/28 2005 vintage



Source: OFHEO, LoanPerformance

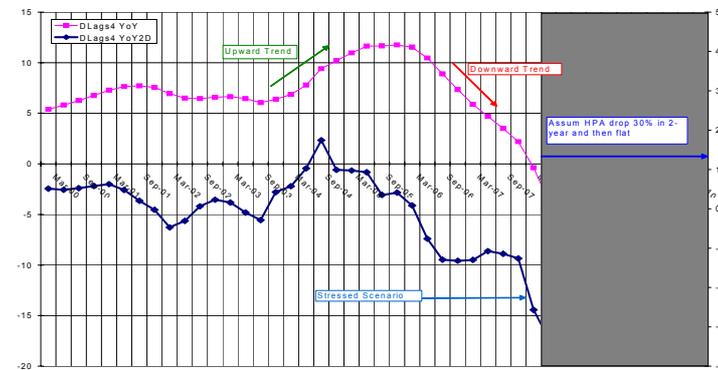
<sup>10</sup> The Housing Price Index HPI used in this paper is published by Office of Federal Housing Enterprise Oversight (OFHEO) as a measure of the movement of single-family house prices. According to OFHEO, The HPI is “a weighted, repeat-sales index, meaning that it measures average price changes in repeat sales or refinancing on the same properties”. See website of OFHEO [www.ofheo.gov](http://www.ofheo.gov) for details.

## Multi-dimension HPI Impacts

To systematically identify the impacts of HPA, we measure HPA in three aspects regarding each loan:

- Cumulative HPI, an accumulative HPA since origination, is calculated based on HPI levels to capture equity gain for borrowers.
- HPA, the change rate of HPI, captures the pulse of housing market.
- HPA2D, the change of HPA, is used to capture the trend/expectations of housing market.

Figure 24. HPA versus HPA2D, Actual and Extreme Simulation



Source: OFHEO, LoanPerformance

The HPA factors form multi-dimension impacts to reflect loan’s up-to-date capital structure, current housing market conditions, and future housing market prospects. We embedded the ‘Cumulative HPI’ into CLTV to build a dynamic CLTV to reflect the dynamic equity value to the property. In a risk neutral analysis, an option model can be easily applied to project the default probability. HPA is already a leading market indicator in explaining defaults. HPA2D basically serves as the second derivative of HPI; it allows us to capture the general expectation on home price movements and market sentiment.

The negative impact due to HPA2D in the 3rd quarter of 2006 is apparently different from the 3rd quarter of 2003 even the HPA numbers are at the similar level.<sup>11</sup> HPA2D undoubtedly offers another dimension that reflects consumers expectations about the general housing market. When HPA2D is negative, the probability of borrowers holding negative equity increases.

The remaining challenge lays in the deterioration of the housing market which is producing unseen record-low HPI levels. While the HPA continues decreasing, HPA2D plunges even faster. Our multi-dimensional HPA empirical fitting merely relies on a very limited range of in-sample HPA data. To extrapolate HPA and HPA2D requires numerous possible market simulations to induce a better intuitive sense of numbers. The shaded area in Figure 24 draws a sample of simulated extreme downturn housing market that assumes a 30% drop of HPI level based on 4<sup>th</sup> quarter of 2007 level and then a leveling-off. Based on the simulation results, the HPA2D starts to pick up at least one-quarter earlier than HPA and one-year earlier than HPI level. While a two-year HPI downturn is assumed, the consumer’s positive housing market expectation reflected in HPA2D effectively reduces their incentive to walk away from their negative equity loans.

<sup>11</sup> We have smoothed HPA and HPA2D series to create better trend lines. A linear weighted distributed lags of last four quarters are adopted for smoothing the series.

This simulation case example clearly shows how the forecasted HPA and HPA2D numbers could provide a better intuitive market sense to the model.

The relationship between HPI and consumer behavior that forms the HPI impact to default and prepayment are then modeled. We illustrate the multi-dimensional HPI impact through an example as below,

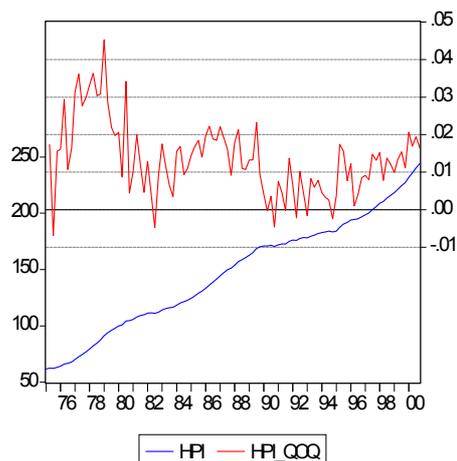
1. HPCUM ↓ (below 5%) => CLTV ↑ => MDR ↑ , SMM ↓
2. HPA ↓ (below 2%) => MDR ↑ , SMM ↓
3. HPA2D ↓ (below -5%) => MDR ↑ , SMM ↓

### **HPI and DPI**

When the economy is experiencing a potentially serious downturn, generating HPI predictions going out three years is a much better approach than random simulations. Since HPI has increased so rapidly since 2000, the current fall could be merely an adjustment to the previously overheated market. The magnitude and ramp up period of the adjustment nevertheless determines a consumer's behavior of exercising their mortgage embedded options. Finding a long-term growth pattern of HPI thus becomes very vital for predicting and simulating future HPI numbers.

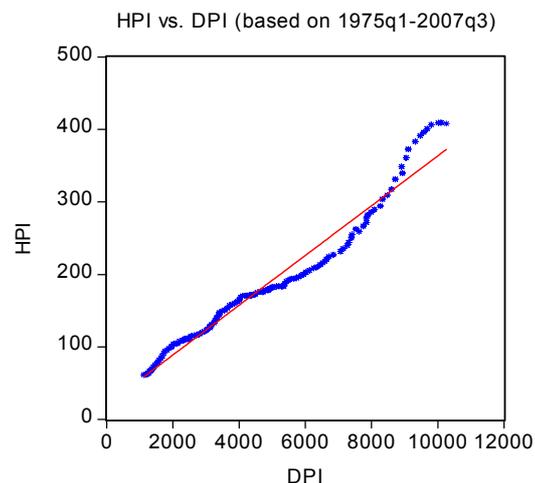
Based on Figure 26, HPI draws a constant relationship with Disposable Personal Income (DPI) in the long run. Since DPI is a more stable process, a long term pattern HPI prediction based on the observed relationship between DPI and HPI provides a better downturn average number. Based on our long-term HPI prediction, HPI could potentially drop as much as 14% by the end of 2009.<sup>12</sup>

Figure 25. HPI & HPA QoQ 1975-2007



Source: US Bureau of Census

Figure 26. HPI vs. DPI



Source: US Bureau of Census

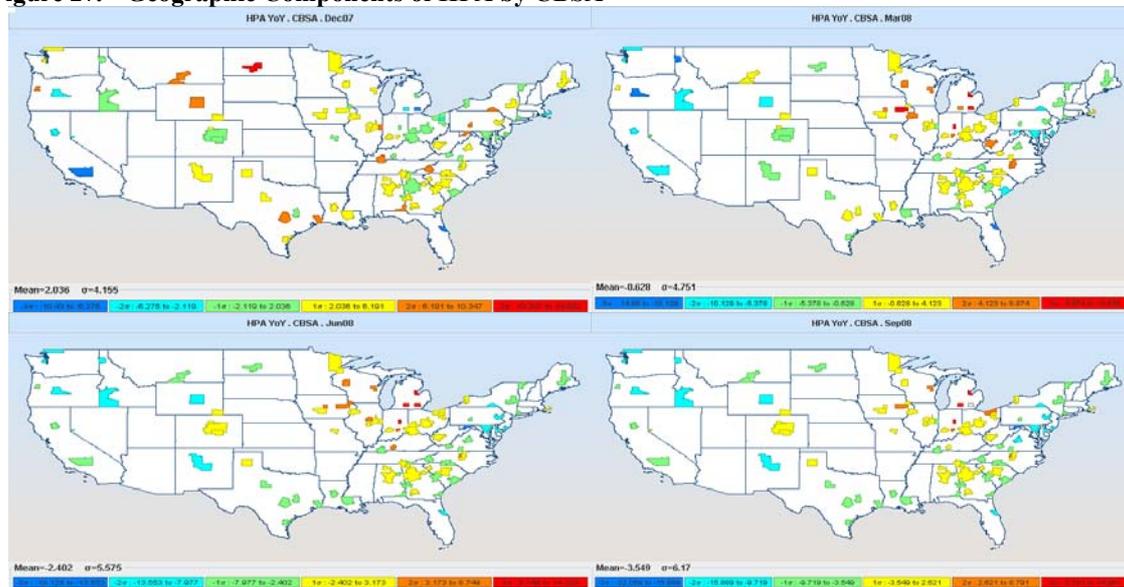
<sup>12</sup> A 5% decrease by the end of 2009 in average plus another 9% based on two standard errors of regression of HPI on DPI result.

### **Geographical location and Local HPI**

In housing market, geographical location (location, location, location or L<sup>3</sup>) is undoubtedly the most important price determinant, as it is globally unique.. While we are pointing out all HPI impacts in general, HPI in the national level does not reflect the actual local situation and thus distorts the default impact ignoring the granularity of detailed local housing market information. The consequence of ignoring this kind of granularity can be very severe when a geographically diversified mortgage pool’s CLTV has a fat-tailed distribution in its high CLTV end. Since the detailed local HPI can vary from the national HPI, loans with negative equity have a higher level of relevance than the use of the national HPI.

Fortunately, we are able to differentiate HPI impacts by drilling down to the HPI information on a state as well as CBSA level. Figure 27 shows the examples of actual levels of HPA on December 2007 and our projection of HPA for June 2008 detailed by CBSA. We started with a national level HPI model to obtain the long term relationship between HPI and DPI. We then build dynamic correlation matrix between national and state as well as national and CBSA levels respectively that dynamically estimates parameters and generate forecasts on the fly. The CBSA level HPI is especially important for calculating dynamic CLTV. Since the cumulative HPI (HPCUM) is calculated as the cumulative HPA since origination to capture wealth effect for generating dynamic CLTV. The more detailed level information apparently helps to predict if a mortgage has crossed the negative equity zone.

**Figure 27. Geographic Components of HPA by CBSA**



## PREPAYMENT MODELING

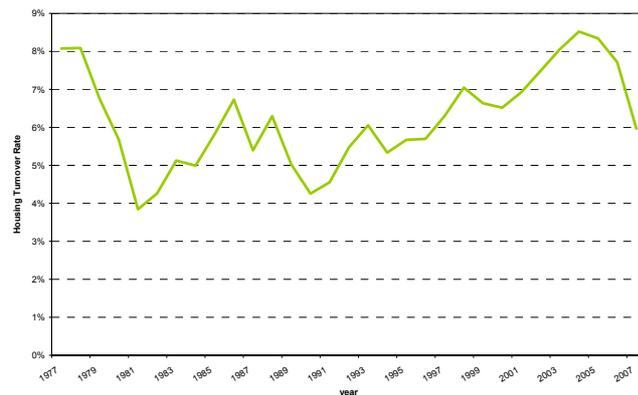
### **Prepayment Modeling Factor Components:**

Housing Turnover and Age  
Refinancing  
Teaser Effect  
Interest Only (IO) Effect  
Burnout Effect  
Seasonality  
Loan-To-Value Effect  
Credit Score FICO Effect  
Prepayment Penalty  
Housing Price Wealth Effect

### **Housing Turn Over and Seasoning**

Housing Turnover rate is the ratio of total existing single-family house sales over the existing housing stock.<sup>13</sup> With the exception of cases in early 80s, the housing turnover rate has been rising steadily for the last fifteen years until 2005. The result of a rising housing turnover rate indicates that home owners are capable of moving around more than in the past. In the housing market boom era, it also indicates the height of speculation. When the housing boom came to an end, the housing turnover rate started to reduce. The movement of housing turnover after 2005 shows exactly the same directionality. Since the housing turnover rate used as the base prepayment speed and could generate a significant tail risk of principal loss given the same default probability, it is especially crucial for a high default and slow prepayment environment like the current one.

Figure 28. U.S. Housing turnover 1977-2007



Sources: National Association of Realtors and Beyondbond

<sup>13</sup> We use five year moving average of 'Total Occupied Housing Inventory' based on U.S. Census Bureau times 0.67 to estimate the total Single-family Housing Stock.

## Seasoning

The initial origination fee and the loan closing expenses usually takes a few years to be amortized, and this discourages the new mortgagors from prepaying their mortgages early in the mortgage term. This ramping-up effect is the seasoning factor. Figure 29 shows the age pattern observed for Fixed Rate loans. The ramping-up period initially lasts for the first few months and then it starts to level off or decrease due to other prepayment factors.

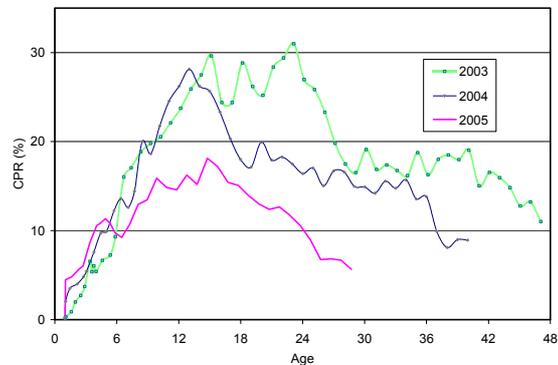
Hybrid ARMs exhibit similar patterns initially during the first 12 months. For hybrid like ARM 2/28, the prepayment level climbs up from 0% to around 20-50% CPR within the first 12 months. After that, the acceleration of the prepayment levels starts to slow down until right before the teaser period ends. The difference in prepayment levels can be readily observed after the 12th month when shorter Hybrids begin to show higher prepayment rates. The reason why ARM 2/28 borrowers show higher prepayment levels can be due to the faster housing turnover of the hybrid group. After the first 12 months, the prepayment generally stays around the same level with a wave-like trend peaking around every 12 months. The seasoning pattern is illustrated in Figure 30.<sup>14</sup>

## Teaser Effect

The teaser effect is the most distinctive feature of Hybrid ARM products. We define the term as the behavior that tends to persist right around the first reset where borrowers seek alternatives to refinance their mortgages or simply prepay them to avoid higher interest rates. In the following section we will describe the empirical statistics gathered to support the teaser effect.

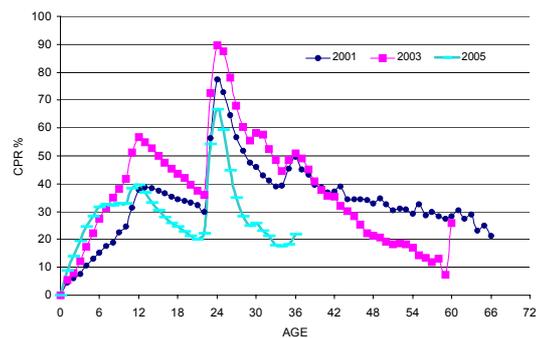
Approximately one to two months before the end of the teaser period, a sharp rise in prepayments occurs. The effect is apparently larger for shorter Hybrids like ARM 2/28 since shorter Hybrids are exposed less to other prepayment factors such as refinancing

Figure 29. CPR over various vintages of Discount Fixed Rate, coupon=6%



Sources: Beyondbond, Loan performance

Figure 30. CPR over various vintages of ARM2/28



Sources: Beyondbond, Loan performance

<sup>14</sup> For the data pooling in terms of its vintage year, we usually use the loan distribution data for grouping information. It helps to maintain the relationship while examining the relationship with macro-economic variable for time series analysis. It however distorts the age pattern since the loans within same vintage year could be underwritten in different months. The seasoning graph is specifically grouped by the loan's seasoning age to better understand the age pattern.

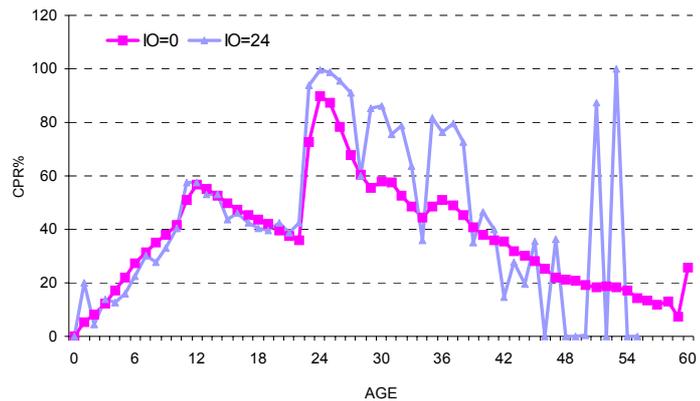
and burnout before the teaser period. The peak level is reached just about two months after the teaser period ends. Teaser impact usually observed as a sudden jump in prepayment levels. This spike happens whenever borrowers are able to refinance with lower cost alternative.

### Interest Only (IO) Effect

During the teaser period before IO period, the prepayments of ARM 2/28 with or without IO track each other fairly well. Before the end of the teaser, loans with IO exhibit higher prepayment level than the regular ones. IO borrowers are even more sensitive to the payment level since they are paying only the interest portion before the teaser. Once the teaser ends, they will start to pay not only higher interest but also an additional amount of

amortized principal. Their incentive to refinance is definitely higher than regular ARM 2/28 borrowers. Even worse, if they can not find a refinancing alternative, they could face affordability issues and increased default risk. We will address this more in the interaction between prepayment and default section.

Figure 31. CPR over various vintages of ARM2/28

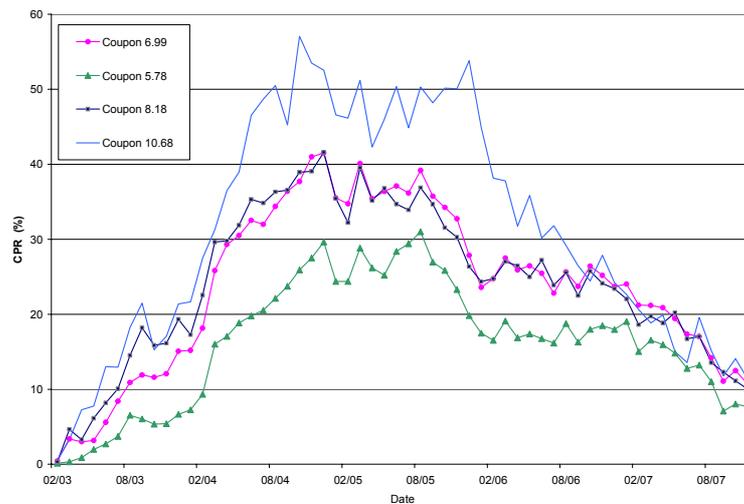


Sources: Beyondbond, Loan performance

### Refinance

The prepayment incentive is measured as the difference between the existing mortgage rate and the prevailing refinancing rate, which is commonly referred to as the refinance factor. As the refinancing factor increases, the financial incentive to refinance increases and thus changes prepayment behavior. When the loans are grouped by their coupon rates during the teaser period, the differences of

Figure 32. Refinance: Stratification by Coupon, Fixed Rate, 2003 vintage



Sources: Beyondbond, Loan performance

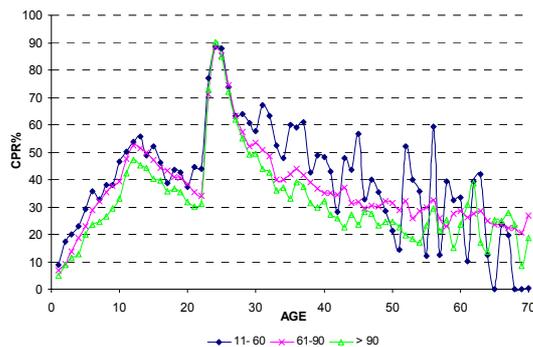
prepayment levels are quite apparent. They behave in similar patterns but loans with higher coupons tend to season faster due to the financial incentive to refinance while loans with lower rates tend to be locked-in as the borrowers have secured the lower rates.

### **Burnout Effect**

The heterogeneity of the refinancing population causes mortgagors to respond differently to the same prepayment incentive and market refinancing rate. This phenomenon can be filtered out as the burnout. The prepayment level usually goes up steadily with occasional exceptions across the high financial incentive region. The major reason for this is due to the burnout phenomena in which borrowers that have already refinanced previously and have taken advantage of the lower rates and are less likely to refinance again without additional financial incentives. To capture such a path-dependant attribute, our prepayment model utilizes the remaining principal factor to capture the burnout effect in order to reduce the chances of overestimating the overall prepayment levels.

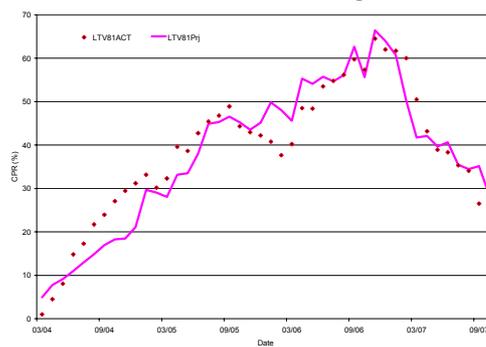
### **CLTV Wealth Effect**

Figure 33. Wealth: CPR over various CLTV of ARM2/28



Sources: Beyondbond, Loan performance

Figure 34. Fitted CPR over CLTV 81-90 of ARM2/28, 2004 vintage



Sources: Beyondbond, Loan performance

As a property’s price appreciates, the LTV of a loan gradually decreases. Borrowers with a low LTV may be able to refinance with a lower interest rate. Some borrowers may even find themselves in an in-the-money situation where they can sell their property to make an immediate profit. Historically, home prices continue to increase with age, and more and more loans will fall into this “low LTV” category which has an increasing likelihood of prepayment. We use a combination of CLTV, HPA and age to model this effect.

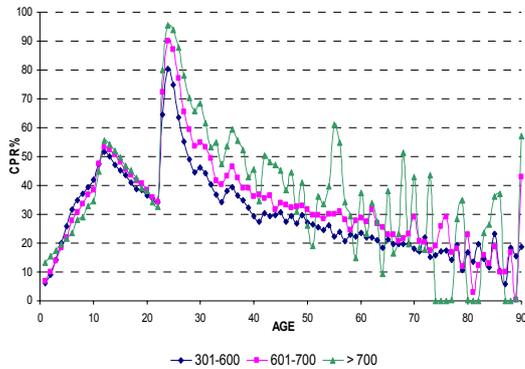
### **FICO Credit Effect:**

Subprime market consists of people with limited credit history and/or impaired credit score. The high FICO score group usually is offered more alternatives to refinance and thus has the flexibility to choose between different products. For those people who are on the threshold of subprime and prime market, they could be upgraded to participate in the prime market during the course of the loan life. Thus the prepayment is an increasing monotonic function with respect to FICO.

We can see a combined effect of FICO and CLTV on CPR. Those people who have a low CLTV and a high FICO score can easily refinance and will have highest prepayment rate;

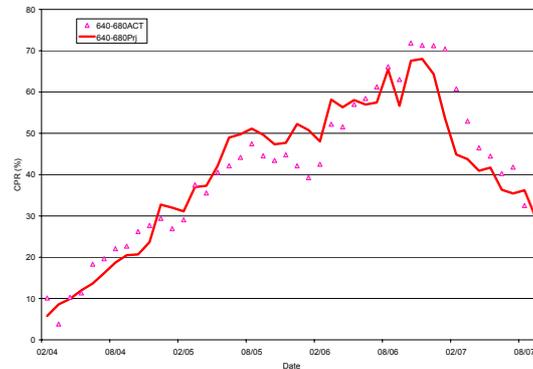
while people who have high CLTV and low FICO score will be on the side of the pendulum with lowest prepayment rate. Figure 36 gives a sample CPR fitting result based on ARM 2/28, 2004 vintage pools.

**Figure 35. Credit: CPR by FICO of ARM2/28**  
2000 to 2004 vintage, CLTV 70-90, DTI 35-45



Sources: Beyondbond, Loan performance

**Figure 36. Credit: Fitted CPR**  
FICO 641-680 of ARM2/28, 2004 vintage



Sources: Beyondbond, Loan performance

## Prepayment Penalty

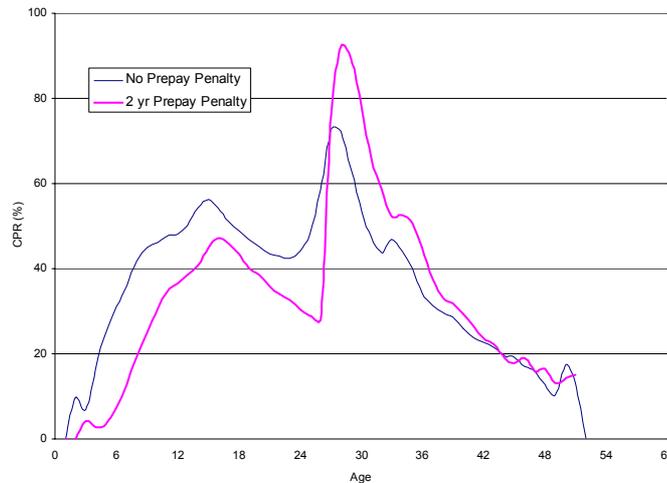
A prepayment penalty fee in the loan structure is no doubt a negative incentive and deters prepayment.

Prepayment is in essence an embedded call option with remaining balance as its strike price for the option. The penalty simply adds to that strike price as additional cost when borrowers exercise the option. That ad hoc additional cost will be reduced to zero after the penalty period.

Figure 37 shows the prepayment difference when a penalty clause is in place.

Before the 2-year penalty term, prepayment is consistently slower than no penalty loans. Soon as the penalty period ends, prepayments surge dramatically and surpasses the no penalty loans within 3-months and then consistently maintain a faster prepayment speed.

**Figure 37. CPR over various vintages of ARM2/28**



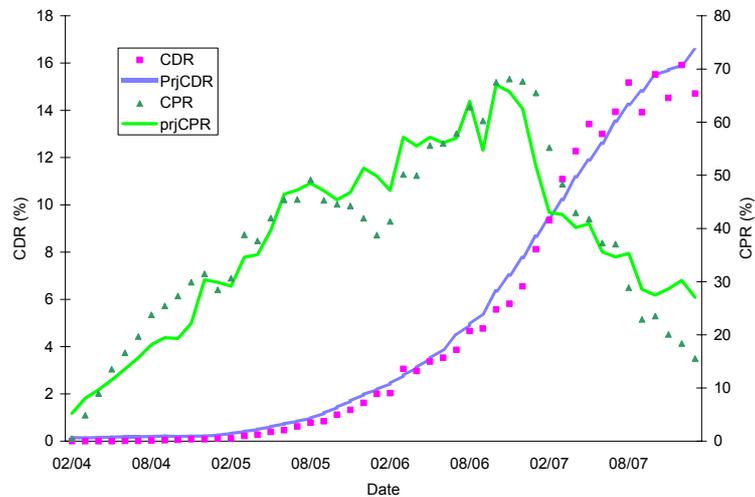
Sources: Beyondbond, Loan performance

## Interaction between Prepayment and Default

As we stated in the beginning of the model framework, prepayment is a call option and default is a put option with its loan balance and collateral value as its strike price respectively. A borrower will continuously find incentives to exercise it if the option is in-the-money.

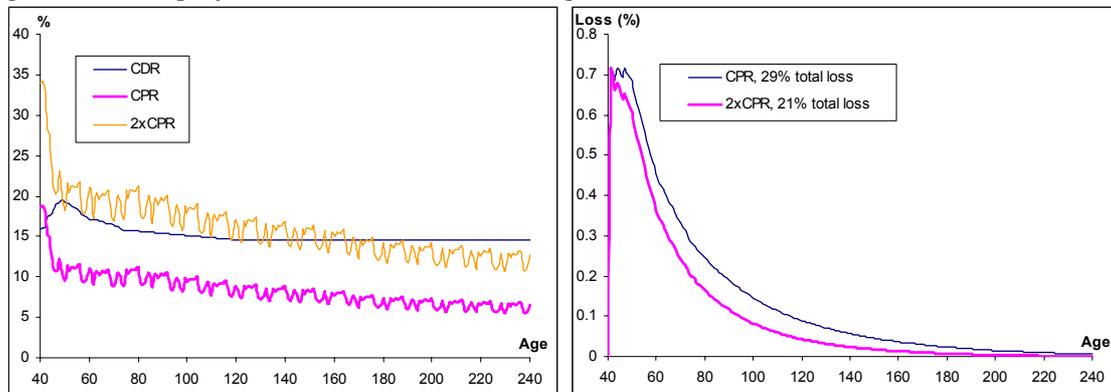
When we estimate the prepayment and default for a pool of mortgages, the remaining principal factor encompasses the entire history of the pool’s prepayment and default rates. Since estimating losses is the main focus for modeling default and prepayment, it is of particular importance in a slow prepayment environment. Given the same default probability, the tail risk to the loss curve will still increase substantially. Figure 39 presents a tail risk example. When the prepayment speeds double, the total loss increases from 21% to 29% given the same default speeds.

Figure 38. CDR and CPR of ARM2/28, 2004 vintage



Sources: Beyondbond, Loan performance

Figure 39. Loss projection of ARM2/28, 2004 vintage



Sources: Beyondbond, Loan performance

Because the history of prepayment and default rates can seriously affect the remaining principal factor for any given pool of loans, tracking and rolling the principal factor for loan pool is one of the most important factors for the model projections and future forecasts. Prepayments are specified prior to defaults and are removed from the outstanding balance and are therefore not available to default in the future.

## DELINQUENCY STUDY

### *Delinquency, the leading indicator*

Is delinquency a good leading indicator for default? When a borrower is late for his payment for more than thirty days, a 30-day delinquency is reported. If the late payment exceeds two month, a 60-day delinquency is reported. After 90-day delinquency, a loan will start its foreclosure process depending on the judicial status of each state and is considered to be in default. Since a default is a consequence of delinquency, the spectrum of delinquencies should be leading indicators of future defaults. We should be able to simply roll delinquency numbers month to month into actual defaults. The question is whether there is a constant relationship that can be parameterized or not. The time series plots of defaults and the spectrum of delinquencies for 2003 vintage are shown in Figure 40. The cross correlations indicate an approximately six-month period for a 30-day delinquency manifest into default as shown in Figure 41.

Figure 40. Default and Delinquency over time for 2003 vintage

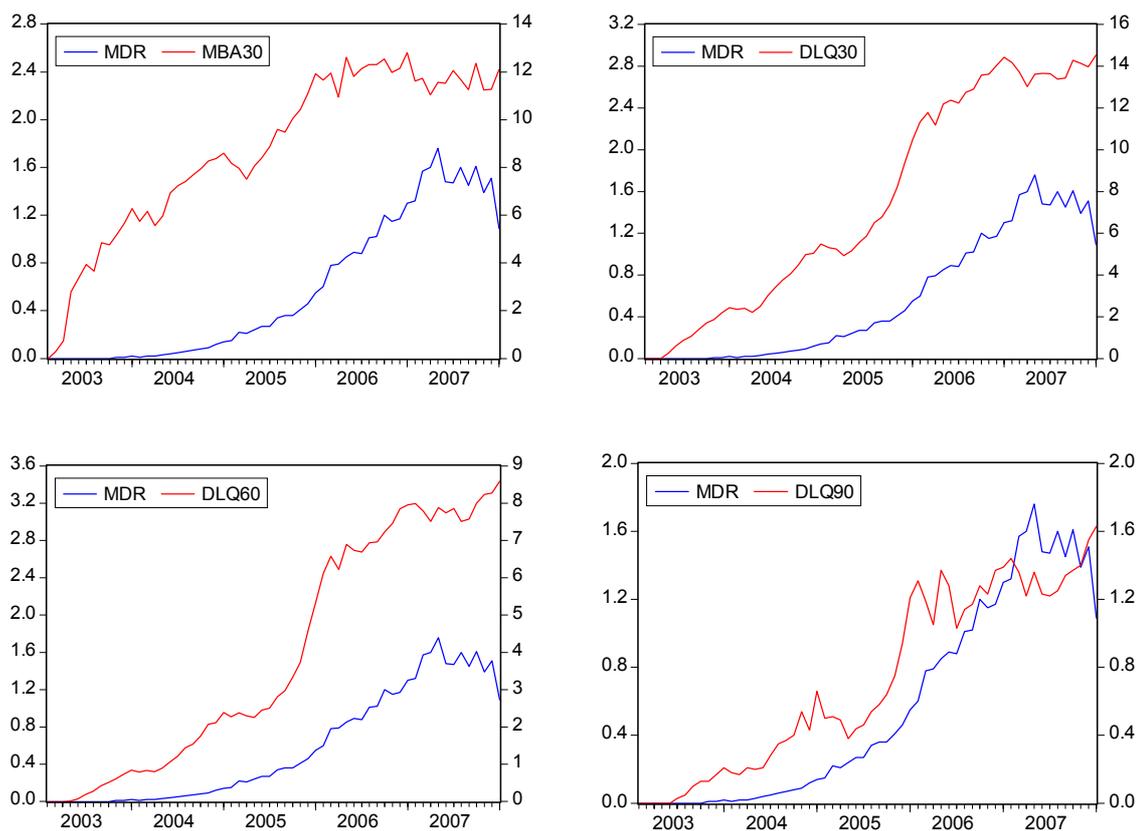
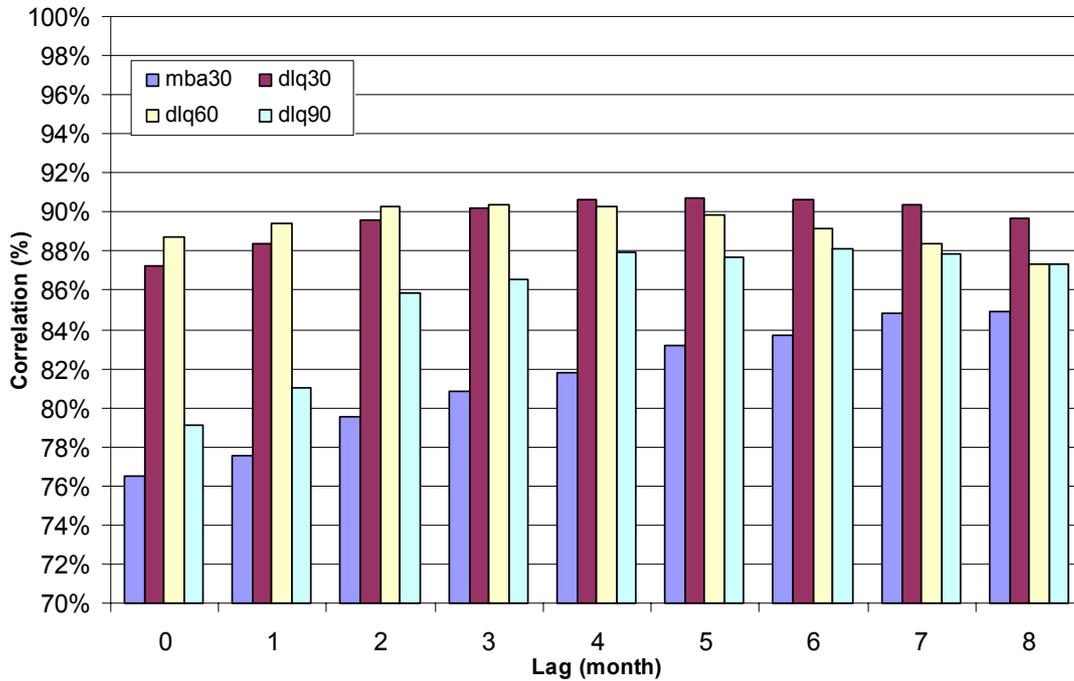


Figure 41. Cross Correlations of Default and Delinquency for 2000 vintages



### Analysis among delinquency spectrum

Figure 42. Correlations between various Delinquencies

	mba30	dlq30	dlq60	dlq90
mba30(-1)	0.974228	0.896283	0.849914	0.819303
dlq30(-1)	0.892006	0.99476	0.989324	0.931421
dlq60(-1)	0.842606	0.980814	0.993112	0.915923
dlq90(-1)	0.8199	0.937639	0.934675	0.898144

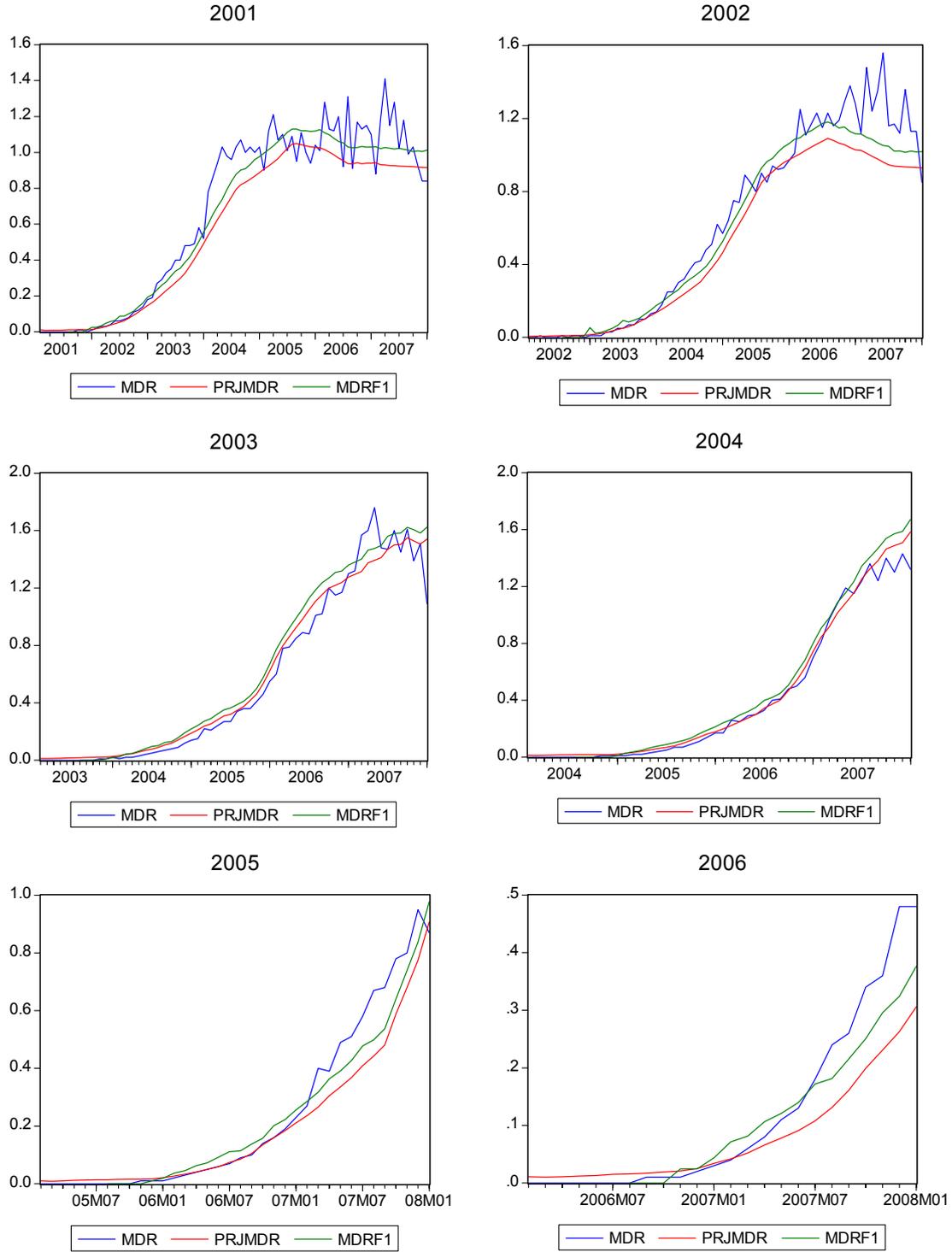
The results among delinquency spectrum show a very significant cross correlation between delinquency and it's lagged earlier tenor.

### A Delinquency Error Correction Default Model

Based on the results shown previously, the spectrum of various delinquencies provides a good indication and can be parameterized for near-term projections. The benefit of including delinquency to project defaults is that it does not require specific consumer behavior theory to be applied. By simply looking at delinquency report, we are able to project the likelihood of defaults. It however, suffers from the long term view that if a loan fundamentally carries lower credit-worthy characteristics such as low CLTV it has a propensity to default. We however are impressed with their short-term forecast ability. In order to fully utilize the information provided by delinquency and the econometric model based on consumer behavior theory, we have integrated both and created a delinquency error correction model.

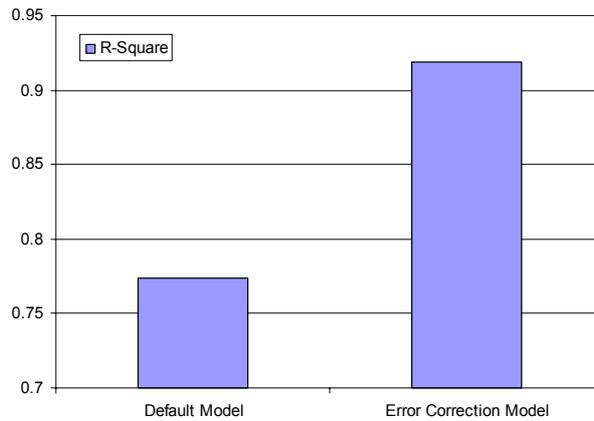
The fundamental idea is that not only can the long-term view and various scenarios based on changing view of macro-economic variables be adopted, but also the immediate/early warning signs from delinquency can be observed and utilized.

Figure 43. Delinquency Error Model: Actual vs. Fitting



In our error correction model, we start by projecting default rate via using the default function with fitted parameters. We then layer on a 6-month lagged 30-day delinquency as an additional exogenous variable to regress the fitted errors. The process is then repeated with adding 5-month lagged 60-day and 4-month 90-day delinquency rates as new regressors respectively. The results are very encouraging when compared to the base model without error correction. The additional  $R^2$  pick-up is around 15%.

**Figure 44. Comparison of Model Explanation Power for 2000-2007 Vintages**



Sources: Beyondbond, Loan performance

The additional  $R^2$  pick-up is around 15%.

## **CONCLUSION**

### ***Why Innovate?***

Traditionally, practitioners have observed consumer behavior through historical defaults and prepayments while building an econometric model with several quantifiable factors. These factors include seasoning patterns, underlying loan characteristics, such as mortgage coupon, FICO score, loan-to-value, and debt-to-income ratio, and macro-economic variables, such as prevailing mortgage rate, housing price appreciation. In order to fit the historical data, non-linear functions are usually constructed with parameters around the factors to explain default and/or prepayment probabilities. During the process of historical sample fitting to the econometric model, the traditional modelers usually miss the following:

1. Traditional models focus on fitting in-sample data with a unique parameter set by vintage. Although the in-sample data fitting provides a much easier fit of the parameter set, it assumes that borrower's behavior varies given same loan characteristics and loan age. It creates a disconnect among vintages and cannot be simply applied to new loans.
2. Borrower behaviors underlying, LTV, FICO, and DTI were implicit, but not fully quantified in dynamic form by traditional models. Since loan information such as LTV, FICO, and DTI levels are not periodically updated after the loan origination date, the accuracy of projecting performance of seasoned loans diminishes as time passes as it is based on original loan information.
3. Out-of-sample projections may produce counter-intuitive results. Since macro-economic variables, such as HPA, Unemployment, Personal Gross Income future, etc. can be very important factors for in-sample fitting, they however, do not provide insight for new scenarios. If a new scenario has not occurred historically, a stress test for the new scenario should be thoroughly pre-examined.
4. Traditional models focus at the national level rather than drill down to local housing markets. Since housing prices are highly dependent on its location, a model with more detailed housing information can make a dramatic difference to its forecast accuracy.
5. Traditional models treat prepayment and default independently, it ignores the complexity and interaction between put and call options. For example, prepayments slow down substantially (burnout) when the principal factor reduced.
6. Traditional models do not dynamically quantify feedback from other leading indicators such as delinquency rates.

Because of the credit crisis, we now know we must have missed something in the traditional models. It required us to take a hard look at the models and methodologies employed today and see what was needed to provide a better interpretation of the data and current conditions..

## **Innovation**

Having addressed the pitfalls that traditional models fail to address, we have built a Dynamic Econometric Loss (DEL) model framework with the following innovations:

Consistent parameter set for all vintages via the addition of consumer behavior factors.

1. Dynamic consumer behavior factors
  - a. CLTV ratio (via cumulative HPA since origination) which reflects housing market wealth effects during housing boom/bust eras.
  - b. DTI ratio (via unemployment rate forecasts) which addresses housing affordability.
2. Complete study of HPA index prior to model-fitting
  - a. HPCUM as the cumulative HPA since origination to capture wealth effect.
  - b. HPA to capture the pulse of the housing market.
  - c. HPA2D as the change of HPA to capture the trend of the housing market. HPA2D successfully captures the timing of the defaults for 2005 to 2006 vintages.
  - d. In-sample and out-of-sample HPA fit testing to ensure the model's robustness.
3. A CBSA detailed level HPA model allows us to not only better understand local housing markets, but also generate more precise projections.
4. Recursive calculations along seasoning paths while estimating/projecting prepayments and defaults.
5. An error correction model that systematically builds the linkage between delinquency and default which enhances our default forecast.

## **Advantages**

The implementation based on our model framework will capture the loss pattern during the recent period but can also forecast future prepayments, defaults and losses based on various macro-economic market scenarios. The implementation advantages are:

1. Multiplicative and additive factors for each non-linear function (boot-strapping Maximum Likelihood Estimation)
2. Comprehensive consumer behavioral economic theory applied in practice
  - a. Develop a consumer behavior based economic theory.
  - b. Estimate consumer behavior via an econometric model.
  - c. Apply the econometric model to prepayment and default.
3. Fully utilize HPA time-series information
  - a. A built-in time-series fitting model that dynamically estimates parameters and generates forecasts on the fly. For example,

- i.  $HPCUM \downarrow$  (below 5%)  $\Rightarrow$   $CLTV \uparrow \Rightarrow$   $MDR \uparrow$ ,  $SMM \downarrow$

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- ii. HPA ↓ (below 2%) => MDR ↑ , SMM ↓
  - iii. HPA2D ↓ (below -5%) => MDR ↑ , SMM ↓
4. Multiple built-in time-series fitting models at the national, state, and CBSA level that dynamically estimate parameters and generate forecasts on the fly.
  5. Built-in recursive calculator along seasoning paths for projecting prepayments and defaults.
  6. A set of error correction fitting models that estimate parameters within the spectrum of delinquencies and defaults that are generated on the fly.

## **Findings**

In order to understand how a loan prepays or defaults, we investigate consumer behavior via loan characteristics utilizing static factors and relevant macro-economic variables as dynamic factors. For each factor, we have constructed a non-linear function with respect to magnitude of the factor. We then built the default/prepayment function as the linear combination of all factors to justify the impact of each factor accordingly. Since a loan can either prepay or default over time, we then continue to ensure that the principal factors are rolled properly for prepayment and default forecasts.

During the fitting, a list of interesting findings were noted:

When the level of HPA is considered the main blessing/curse for the rise and fall of subprime market, we find that cumulative HPA and the change of HPA contribute additional dimensions to effect prepayment and defaults.

1. HPI is significantly correlated to DPI over a long-term period. Since DPI is a more stable time series, it suggests that HPI will eventually adjust to coincide with DPI growth rate.
2. Default is strongly correlated to the spectrum of delinquency rates. By applying the fitted parameters between default and delinquency rate to an error correction model is able to effectively improve default predictability.

## **Future Improvements**

So far, our model provides us a set of better tools to explore consumer behavior and various impacts due to selected macro-economic variables as dynamic factors and thus project default and prepayment probabilities in a precise and timely manner. Nevertheless, modeling the embedded mortgage options for default and prepayment is an on-going learning process. While we are encouraged by our findings, there is a myriad of new questions for us to address, with an aim to continuously improve and fine tune the model in the future. Some example areas of further investigation are briefly described below.

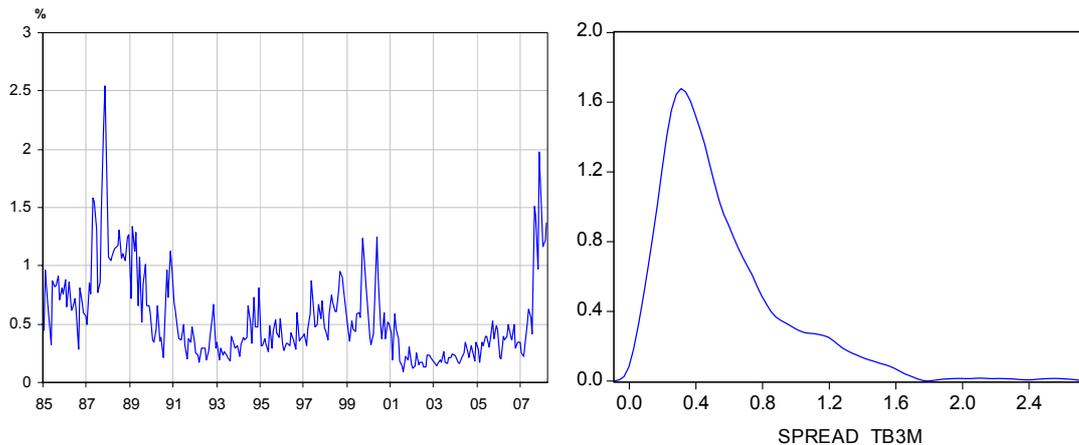
### **Business Cycle – Low Frequency of Credit Spread**

While studying the dynamic factors in the Default Modeling section, we focused mainly on the HPI impact on consumer behavior and introduced the DPI as another macro-economic variable to determine the long-term growth of the economy. In the beginning of this paper, we were wondering how a relatively small volume of loans could result in a

subprime crisis that proved to be detrimental to the entire U.S. financial markets and global financial system. We believe that the subprime crisis is merely the tipping point of unprecedented credit market easing since early this century. During the extreme credit ease era, yield hungry investors needed to enhance their returns through investment on either highly leveraged securities or traditionally highly risky assets such as subprime loans. Through rapid growth of the credit default swap in derivative markets and RMBS, ABS, and CDOs in the securitization markets, subprime mortgage origination volume reached record highs beginning after year 2003. The credit ease impacted not just the subprime market. All credit based lending from credit cards to auto loans, and leverage buy-out loans were enjoying a borrower friendly lending environment as lenders went on a lending spree. While the credit default rates reached their historical low in last decade and resulted in extremely tight spreads among credit products, a longer view of the history of business cycles started to reveal warning signs of the potential downside risk.

For example, the TED Spread dramatically widened after August 2007 which was a re-occurrence of the late eighties market environment. Over the past 20 years, traditional calibration models only focused on shorter time frames have missed the downside “fat-tail”. The improbable is indeed plausible. Is there a better method to mix the long-term low frequency data with their short-term high frequency data and then provide a better valuation model?

**Figure 45. Historical TED Spread and Histogram**



Source: Beyondbond, Inc.

### **Dynamic Loss Severity**

Traditionally, prepayment and default modeling is the main focus of the fundamental research for mortgages while loss severity and timing lags for loss recovery is simply run as a given. The detailed HPA information provided at the CBSA level and better detailed information provided by Servicers in recent years has allowed us to create a more robust dynamic loss severity estimation and we will continue work with Servicers to develop improved estimates in the future.

## **APPENDIX I DEFAULT AND PREPAYMENT DEFINITION**

We consider a loan to be in default if it meets both of the following criteria:

- 1) The loan is not able to generate any future investor cashflow
- 2) The loan has been in foreclosure, REO or reporting loss in prior reporting period

The Monthly Default Rate (MDR) is defined as the percentage of defaulted amount as a sum of all default loan balance compared with the aggregate loan balance of that period.

SMM (Single Month Mortality) is calculated by formula:

$$SMM = \frac{\text{Scheduled Balance} - \text{Current Balance}}{\text{Scheduled Balance}}$$

If we have MDR and SMM, then we can simply derive CDR and CPR from them by using the formula:

$$CDR = 1 - (1 - MDR)^{12}$$

$$CPR = 1 - (1 - SMM)^{12}$$

## APPENDIX II GENERAL MODEL FRAMEWORK

$$\begin{aligned}
 y_t^{(s)} &= \sum_{k=0}^K \varphi_k(X_t^{(k)} | \alpha_m^{(k)}, \beta_m^{(k)}; m \in [0, M^{(k)}]) \cdot \\
 &\quad \prod_{i=0}^I \lambda_i(X_t^{(i)} | \alpha_m^{(i)}, \beta_m^{(i)}; m \in [0, M^{(i)}]) \cdot \\
 &\quad \prod_{j=0}^J \eta_j(X_{t,m}^{(j)} | \alpha_m^{(j)}, \beta_m^{(j)}; m \in [0, M^{(j)}]) \\
 = &\quad \sum_{k=0}^K \varphi_k(X_t^{(k)} | \alpha_m^{(k)}, \beta_m^{(k)}; m \in [0, M^{(k)}]) \cdot \\
 &\quad \prod_{i=0}^I \lambda_i(X_t^{(i)} | \alpha_m^{(i)}, \beta_m^{(i)}; m \in [0, M^{(i)}]) \cdot \\
 &\quad \prod_{j=0}^J ((1 + \sum_m^{M^{(j)}} X_{t,m}^{(j)} \beta_m^{(j)}) | \alpha_m^{(j)}, \beta_m^{(j)}; m \in [0, M^{(j)}])
 \end{aligned}$$

Where

$y_t^{(s)}$  is an observable value at time t for dependent variable type s

$\varphi_k$  is a spline interpolation function with pair-wise  $(\alpha_m^{(i)}, \beta_m^{(i)})$  knots

$X_t^{(k)}$  is an observable value of factor k at time t

$K$  is the number of additive spline functions

$\lambda_i$  is a spline interpolation function with pair-wise  $(\alpha_m^{(k)}, \beta_m^{(k)})$  knots

$X_t^{(i)}$  is an observable value of factor i at time t

$I$  is the number of multiplicative spline functions

$\eta_j$  is equal to  $(1 + \sum_m^{M^{(j)}} X_{t,m}^{(j)} \beta_m^{(j)})$  and is a linear combination function with multiplier  $\beta_m^{(j)}$  of  $X_{t,m}^{(j)}$ ; where  $X_{t,m}^{(j)}$  is an observable value of the type m factor at time t, while  $\beta_m^{(j)}$  is the composition ratio of the distinct factor j of type m

$J$  is number of linear functions

## APPENDIX III – DEFAULT SPECIFICATION

A whole loan mortgage starts at  $t_0$  and matures by  $t_n$ , its MDR by time  $t$  can be driven by two type of variables – static and dynamic.

Collateral characteristics such as mortgage rate, loan size, IO period, teaser period, loan structure, term to maturity, geographic location, FICO, and CLTV are static factors since their impacts diminish over time while the loan is getting seasoned.

The macro-economic variables over time such as Housing Price Index, mortgage interest rate, unemployment rates, Gross Disposable Income, and inflation rates are dynamic. They are publicly observable and will tune forecast of the default rate based on the scenario assumption.

We formulate our default function MDR as follows:

$$D_t = \varphi_{LTV}(v_t | LTV_j, h_t) + \varphi_{FICO}(c_j) \cdot \lambda_{rate}(r_t | WAC_t) \cdot \lambda_{age}(a_i | a_0) \cdot \lambda_{DTI}(d_j | DTI_j, DOC_j) \cdot \lambda_{IO}(g_t | IO_j, a_i) \cdot \lambda_{size}(s) \cdot \lambda_{HPA}(HPA) \cdot \lambda_{H2D}(H2D) \cdot \eta_{DOC}(Doc_m) \cdot \eta_{LIEN}(LIEN_m) \cdot \eta_{PURPOSE}(PURPOSE_m)$$

### Where

$\varphi_s$  are spline functions in MDR % and are additive to form a base value

$\lambda_s$  are spline functions as multipliers for the MDR adjustments

$v_t$ : CLTV by time  $t$  where initial CLTV is assumed at time  $t_0$

$r_t$ : Ratio spread of  $WAC_t$  over original WAC rate

$c_j$ : FICO score of loan  $j$

$a_i$ : Age of loan  $j$

$d_t$ : DTI

$g_i$ : Remaining IO period if IO exists and is positive

$l_j$ : Size of loan  $j$

$\varphi_{LTV}$ : Original LTV level &  $HPA_t$

$$v_t = v_t(v_0, h_t, z_j)$$

$H_{t_i}$ : HPI at time  $t_i$  since origination date  $t_0$

$z_i$ : Geographic zip code  $j$ , e.g.  $z_1 = z(\text{CA}) = 1.3$

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$$z_2 = z(\text{OH}) = 1.1$$

$$z_3 = z(\text{MI}) = 1.01$$

$$z_0 = z(\text{Other}) = 1$$

the function form of  $v_t$

$$v_t = \frac{v_0 \cdot H^{t(i-lag)}}{H^{t(0-lag)}} \cdot z_j$$

$h_t$ : the functional form of  $h_t$  as simple AR(2) model

$$h_t = \beta_0^h + \beta_1^h h_{t-1} + \beta_2^h h_{t-2} + \varepsilon_t$$

Where all the parameters can be independently regressed by  $h_t$ 's time series data

$z_j$ : the functional form of  $z_j$  is setup as a dummy variables

$$z_j = \beta_j^z * z_{(j)} \quad \text{if } j = \text{"CA"} \text{ and parameter } \beta_j^z \text{ can be calibrated by default data by}$$

bootstrapping the value

$f_t$ : is the actual principal factor and will be either observed for in-sample filtering or simulated for out-of-sample forecast

**FICO**: to check if credit scores (original) is a good measure of default

$c_j$ : the functional form of  $c_j$  will be a spline (natural, Linear, tension spline) function with fixed FICO locators,  $j$ 's (suggested only)

$$[250, 350, 450, 500, 525, 550, 550, 580,$$

$$600, 625, 650, 680, 700, 720, 750, 800, 820]$$

and parameters can be calibrated for Default data base & fine-tuned

**AGE**: Default probability increases as loan get seasoned but eventually reach to a plateau given others constant

$a_t$ : we will sample linear spline function from 0 to 1 to apply age locators

$$[0, 1, 5, 10, 15, 20, 30, 45, 60, 120]$$

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**DTI Effect:** to check if income level will affect default under assumption of DOC if it's fully available

$$u_t = u_0 \frac{GDP_t}{GDP_0} \cdot \left( \frac{UM_t}{UM_0} \right)^{\beta^{(UM)}}$$

the functional form

$\lambda_u(u_t)$  is a linear spline function of  $u_t$

$$\lambda_{DTI}(u_t, w_j) = (\lambda_u(u_t))^{\lambda_w(w_j)}$$

where

$$\lambda_w(w_0) = 1 \rightarrow \text{Full} = w_0$$

$$\lambda_w(w_1) = 0.1 \rightarrow \text{Low} = w_1$$

$$\lambda_w(w_2) = 0 \rightarrow \text{No} = w_2$$

**RATE Effect**

$$r_t = (WAC_t - MTG_t)$$

$\varphi_{rate}(r_t)$  is a spline function of  $r_t$

- $WAC_t$  is gross coupon which is either observable or can be simulated from index rates & loan characteristic
- Index rates forecasting will be a spread

$$y_{t's} = \beta_0 + \beta_0 y_{t-1} + \beta_1 \text{Swp2}Y_t + \beta_2 \text{Swp5}Y_t + \beta_3 \text{Swp10}Y_t + \beta_4 \text{LIBOR1M}_t + \varepsilon_t$$

for corresponding index rate LIBOR6M, 1Y-CMT, COFI, 5YY – CMT, ...etc.

**IO-Payment-Shock** (to check if surprised payment increase will increase default)

$$g_t = IO_0 - a_t$$

$\lambda_{IO}(g_t)$  is a Linear Spline function of locators [-30, -20, -10, -5, -2, 0, 2, 0, 2, 5, 10, 20]

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**Crowding Out** (to check if the underwriting standard is deteriorated)

$\lambda_{\text{volume}}$  is a Spline function

$vm_t$  is whole loan issue amount ratio (FICO  $\leq 580$ ,  $580 < \text{FICO} \leq 700$ )

\*Note: 30-day Delinquency rate for the (12-month) ratio if delinquency report is available

$\lambda_{\text{size}}$  is a simple step-spline function to if certain loan size after default with locators  
[ $\leq 50k$ ,  $\leq 100k$ ,  $\leq 150k$ ,  $\leq 250k$ , 500k, 800k, 1million]

### **Occupancy**

$\lambda_{\text{ocp}}$  has 3 kinds of occupancy (Owner, Second Home, Investor,)

### **Loan Purpose**

$\lambda_{\text{prs}}$  has 3 kinds of purpose (Purchase, Refi, Cash Out)

### **Lien**

$\lambda_{\text{lien}}$  has 2 lien positions (First lien, Second lien)

### **Loan Document**

$\lambda_{\text{doc}}$  has 3 kinds of documentation type (Full, Limit, and No Document)

## **APPENDIX IV – PREPAYMENT SPECIFICATION**

### **Single Monthly Mortality (SMM) Rate Function**

$$S_t = \varphi_{\text{rate}} (r_t) \cdot \lambda_{\text{turnoverrate}} ( ) \cdot \lambda_{\text{teaser}} (ts_t) \cdot \lambda_{\text{seasonality}} ( ) \cdot \lambda_{\text{cash-out}} ( ) \cdot \lambda_{\text{age}} (a_t) \cdot \lambda_{\text{burnout}} (f_t) \cdot \lambda_{\text{yieldcurve}} ( ) \cdot \lambda_{\text{equity}} ( ) \cdot \lambda_{\text{credit}} ( ) \cdot \lambda_{IO} (g_t) \cdot \lambda_{\text{credit}} (V_t) \cdot \lambda_{\text{issuer}} (IY_{j's}) \cdot \lambda_{\text{size}} (l_{j's}) \cdot \lambda_{\text{penalty}} (N_{\text{yes/no}})$$

#### **Housing Turnover Rate**

Prepayment based on long-term housing turn-over rate that is composed of existing sales over single-family owner's housing stock.

#### **Seasonality**

Monthly seasonality is generally believed to affect prepayments. The belief stems from the mobility of mortgagors, time of housing construction, school year, and weather considerations. For a specific month of the year and ceteris paribus, prepayment rates are directly affected by the related month-of-year's coefficient. Usually, the seasonality pattern tends to be more active in the spring, rises to peak in the summer, decreases

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through the fall, and slows down even more in the winter. The pattern may be different geographically and demographically.

### **Cash-out**

Prepayment is driven by general housing price appreciation.

**Rate Factor**  $\varphi_{\text{rate}}(r_t)$  (to grab REFI-incentive)

$\varphi_{\text{rate}}$  : a natural spline function

20 locators [-10, -5, -2, -1, 0, 0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4, 5, 6, 7, 9, 10, 15, 20]

$$r_t = \begin{cases} \text{WAC} - m_t(\text{Fixed}) \\ \text{WAC}_D - m_t(\text{ARM / Hybrid}) \end{cases}$$

$m_t$  : FH 30-yr/10day commitment rate (FHR3010) as prevailing mortgage rate to measure SATO effect

\*Age Factor: PPY has less incentive due to the consideration of initial financing sunk cost. But the probability increase change time as the 3-yr coast get average out along time.

### **Age**

Mortgages generally display an age pattern.

### **Burnout Effect**

Borrowers don't behave homogeneously while refinancing opportunities appear.

Some are more sensitive than others. If the borrowers are heterogeneous with respect to refinancing incentive, the more interest sensitive group will refinance.

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